A Specification

A.1 Hilberts axioms

Hilbert : THEORY
BEGIN

ASSUMING

Point : TYPE

Shape : TYPE $=$ setof[Point]

Line, Ray, Segment, Angle : TYPE FROM Shape

$\overleftrightarrow{PP'}$: [Point, Point $\rightarrow$ Line]

$\overrightarrow{PP'}$: [Point, Point $\rightarrow$ Ray]

$\overline{PP'}$: [Point, Point $\rightarrow$ Segment]

$\angle PP'P''$: [Point, Point, Point $\rightarrow$ Angle]

$\cong$: [Shape, Shape $\rightarrow$ bool]

LineExists : AXIOM
$\forall\ (A, B :$ Point$) :\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB})$

OnlyLine : AXIOM
$\forall\ (A, B :$ Point$, l :$ Line$) :$
$(A \in l) \wedge (B \in l) \Rightarrow l = \overleftrightarrow{AB} \vee A = B$

ThreeExist : AXIOM
$\exists\ (A, B, C :$ Point$) :\ A \neq B \wedge B \neq C \wedge C \neq A$

TwoPointsOnLine : AXIOM
$\forall\ (l :$ Line$) :\ \exists\ (A, B :$ Point$) :\ A \neq B \wedge l = \overleftrightarrow{AB}$

ThreePoints : AXIOM $\forall\ (l :$ Line$) :\ \exists\ (C :$ Point$) :\ \neg(C \in l)$

SegmentBounds : AXIOM $\forall\ (A, B :$ Point$) :\ (A \in \overline{AB})$

SegmentOnLine : AXIOM $\forall\ (A, B :$ Point$) :\ (\overline{AB} \subseteq \overleftrightarrow{AB})$

SegmentSymmetry : AXIOM $\forall\ (A, B :$ Point$) :\ \overline{AB} = \overline{BA}$

SegmentExtension : AXIOM
$\forall\ (A, B :$ Point$) :$
$\exists\ (C :$ Point$) :\ (B \in \overline{AC}) \wedge C \neq B \wedge C \neq A$

SegmentOrder : AXIOM

$\forall\,(A, B, C:\ \text{Point}):$
$(C \in \overline{AB})\wedge$
$C \neq A \wedge C \neq B \wedge A \neq B \Rightarrow$
$(\neg(B \in \overline{AC}))\wedge$
$(\neg(A \in \overline{BC}))$

TriangleIntersection : AXIOM
$\forall\,(A, B, C:\ \text{Point}, l:\ \text{Line}):$
$(\exists\,(D:\ \text{Point}):$
$(D \in (\overline{AB} \cap l))\wedge$
$D \neq A \wedge D \neq B) \Rightarrow$
$\exists(E:\ \text{Point}):$
$(E \in ((\overline{AC} \cup \overline{BC}) \cap l))\wedge$
$E \neq A \wedge E \neq B$

CongruentSymmetry : AXIOM
$\forall\,(s_1, s_2:\ \text{Shape}):\ s_1 \cong s_2 \Leftrightarrow s_2 \cong s_1$

CongruentExtension : AXIOM
$\forall\,(A, R:\ \text{Point}, s:\ \text{Segment}):$
$\exists\,(B:\ \text{Point}):\ s \cong \overline{AB} \wedge (B \in \overrightarrow{AR})$

CongruentTransivity : AXIOM
$\forall\,(s_1, s_2, s_3:\ \text{Shape}):$
$s_1 \cong s_2 \wedge s_2 \cong s_3 \Rightarrow s_1 \cong s_3$

CongruentCombination : AXIOM
$\forall\,(A_0, B_0, C_0, A_1, B_1, C_1:\ \text{Point}):$
$(B_0 \in \overline{A_0C_0})\wedge$
$(B_1 \in \overline{A_1C_1})\wedge$
$\overline{A_0B_0} \cong \overline{B_0C_0}\wedge$
$\overline{A_1B_1} \cong$
$\overline{B_1C_1} \Rightarrow$
$\overline{A_0C_0} \cong \overline{A_1C_1}$

RayDef : AXIOM
$\forall\,(A, B, C:\ \text{Point}):$
$(C \in \overrightarrow{AB}) \Leftrightarrow$
$(C \in \overline{AB}) \vee (B \in \overline{AC})$

AngleDef : AXIOM
$\forall\,(A, B, C:\ \text{Point}):\ \angle ABC = (\overrightarrow{BA} \cup \overrightarrow{BC})$

CongruentAngle : AXIOM
$\forall\,(\alpha:\ \text{Angle}, A, B, I:\ \text{Point}):$
$\exists\,(D:\ \text{Point}):$
$\alpha \cong \angle ABD\wedge$
$(\overleftrightarrow{AB} \cap \overline{ID}) \neq \emptyset$

OnlyCongruentAngle : AXIOM
$\forall\,(\alpha:\ \text{Angle}, A, B, C, D:\ \text{Point}):$

$\alpha \cong \angle ABC \wedge$
$\alpha \cong \angle ABD \Rightarrow$
$\angle ABC = \angle ABD \vee$
$(\overleftrightarrow{AB} \cap \overline{CD}) \neq \emptyset$

CongruentTriAngle : AXIOM
$\forall\ (A_0, B_0, C_0, A_1, B_1, C_1 :$ Point) :
$\overline{A_0 B_0} \cong \overline{A_1 B_1} \wedge$
$\overline{B_0 C_0} \cong \overline{B_1 C_1} \Rightarrow$
$\angle A_0 B_0 C_0 \cong \angle A_1 B_1 C_1$

ENDASSUMING

LineSym : LEMMA $\forall\ (A, B :$ Point) : $\overleftrightarrow{AB} = \overleftrightarrow{BA}$

SegmentSelfCongruent : THEOREM $\forall\ (A, B :$ Point) : $\overline{AB} \cong \overline{AB}$

SegmentFromRay : THEOREM
$\forall\ (A, B :$ Point) :
$A \neq B \Rightarrow \overline{AB} = (\overrightarrow{AB} \cap \overrightarrow{BA})$

SegmentDivision : THEOREM
$\forall\ (A, C :$ Point) :
$A \neq C \Rightarrow$
$\exists (D :$ Point) : $D \neq A \wedge D \neq C \wedge (D \in \overline{AC})$

NonCongruence : THEOREM $\exists\ (s_1, s_2 :$ Shape) : $\neg s_1 \cong s_2$

END Hilbert

A.2 Lobachevskian geometry

Lobachevsky : THEORY
BEGIN

ASSUMING

IMPORTING Hilbert

Parallel : POSTULATE
$\forall\ (l :$ Line) :
$\exists\ (g, h :$ Line) :
$h \neq g \wedge$
$(g \cap l) = \emptyset \wedge$
$(h \cap l) = \emptyset$

interior : [Point, Angle $\rightarrow$ bool]

interiorDef : AXIOM
$\forall\ (P, A, B, C :$ Point) :
interior$(P, \angle ABC) \Leftrightarrow$

$\neg(B \in \overleftrightarrow{AC}) \wedge$
$(\overline{PA} \cap \overleftrightarrow{BC}) = \emptyset \wedge$
$(\overline{PC} \cap \overleftrightarrow{BA}) = \emptyset$

$\| : [\text{Ray}, \text{Ray} \rightarrow \text{bool}]$

ParallelRay : AXIOM
$\forall\ (P, R, Q, S : \text{Point}) :$
$\overrightarrow{PR} \parallel \overrightarrow{QS} \Leftrightarrow$
$(\overrightarrow{PR} \cap \overrightarrow{QS}) = \emptyset \wedge$
$\forall (D : \text{Point}) :$
$\text{interior}(D, \angle RPQ) \Rightarrow$
$(\overrightarrow{PD} \cap \overrightarrow{QS}) \neq \emptyset$

$\text{directedparallel}((A_0, A_1, B_0, B_1 : \text{Point})) : \text{bool} = (\overleftrightarrow{A_0A_1} \cap \overleftrightarrow{B_0B_1}) = \emptyset \wedge$
$\exists (P, Q : \text{Point}) :$
$(P \in \overleftrightarrow{A_0A_1}) \wedge$
$(Q \in \overleftrightarrow{B_0B_1}) \wedge$
$\forall (S : \text{Point}) :$
$\text{interior}(S, \angle B_1PQ) \Rightarrow$
$(\overrightarrow{PS} \cap \overrightarrow{QA_1}) \neq$
$\emptyset$

ENDASSUMING

ParallelSymmetry : THEOREM
$\forall\ (A_0, A_1, B_0, B_1 : \text{Point}) :$
$\text{directedparallel}(A_0, A_1, B_0, B_1) \Leftrightarrow \text{directedparallel}(B_0, B_1, A_0, A_1)$

END Lobachevsky

B Proofs

B.1 Line symmetry

Verbose proof for `LineSym`.

`LineSym`:

$$\begin{array}{|ll}
 & \\
\hline
\{1\} & \forall\,(A, B:\ \text{Point}):\ \overleftrightarrow{AB}=\overleftrightarrow{BA}
\end{array}$$

Applying OnlyLine
where

`LineSym`:

$$\begin{array}{|ll}
\{\text{-}1\} & (\forall\,(A, B:\ \text{Point}, l:\ \text{Line}): \\
 & \quad (A \in l) \wedge (B \in l) \Rightarrow \\
 & \quad\quad l = \overleftrightarrow{AB} \vee A = B) \\
\hline
\{1\} & \forall\,(A, B:\ \text{Point}):\ \overleftrightarrow{AB}=\overleftrightarrow{BA}
\end{array}$$

Applying LineExists
where

`LineSym`:

$$\begin{array}{|ll}
\{\text{-}1\} & (\forall\,(A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB})) \\
\{\text{-}2\} & (\forall\,(A, B:\ \text{Point}, l:\ \text{Line}): \\
 & \quad (A \in l) \wedge (B \in l) \Rightarrow \\
 & \quad\quad l = \overleftrightarrow{AB} \vee A = B) \\
\hline
\{1\} & \forall\,(A, B:\ \text{Point}):\ \overleftrightarrow{AB}=\overleftrightarrow{BA}
\end{array}$$

Trying repeated skolemization, instantiation, and if-lifting,

This completes the proof of `LineSym`.

Q.E.D.

B.2 Segement self-congruency

Verbose proof for `SegmentSelfCongruent`.

`SegmentSelfCongruent:`

$$\begin{array}{|ll}
 & \\
\hline
\{1\} & \forall\,(A, B:\ \text{Point}):\ \overline{AB} \cong \overline{AB}
\end{array}$$

`SegmentSelfCongruent:`

$$\begin{array}{|ll}
 & \\
\hline
\{1\} & \forall\,(A, B:\ \text{Point}):\ \overline{AB} \cong \overline{AB}
\end{array}$$

Skolemizing,

`SegmentSelfCongruent:`

$$\begin{array}{|ll}
 & \\
\hline
\{1\} & \overline{A'B'} \cong \overline{A'B'}
\end{array}$$

Applying CongruentExtension
where

`SegmentSelfCongruent:`

$$\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(A, R:\ \text{Point}, s:\ \text{Segment}):\\
 & \quad \exists\,(B:\ \text{Point}):\\
 & \quad\quad s \cong \overline{AB} \wedge (B \in \overrightarrow{AR}))\\
\hline
\{1\} & \overline{A'B'} \cong \overline{A'B'}
\end{array}$$

Instantiating the top quantifier in -1 with the terms: A!1
, B!1
, $\overline{A'B'}$
,

`SegmentSelfCongruent:`

$$\begin{array}{|ll}
\{\text{-1}\} & \exists\,(B:\ \text{Point}):\\
 & \quad \overline{A'B'} \cong \overline{A'B} \wedge\\
 & \quad (B \in \overrightarrow{A'B'})\\
\hline
\{1\} & \overline{A'B'} \cong \overline{A'B'}
\end{array}$$

For the top quantifier in -1, we introduce Skolem constants: C!1
,

`SegmentSelfCongruent:`

$$\begin{array}{|ll}
\{\text{-1}\} & \overline{A'B'} \cong \overline{A'C'} \wedge\\
 & (C' \in \overrightarrow{A'B'})\\
\hline
\{1\} & \overline{A'B'} \cong \overline{A'B'}
\end{array}$$

Applying disjunctive simplification to flatten sequent,

`SegmentSelfCongruent:`

{-1}	$\overline{A'B'} \cong \overline{A'C'}$
{-2}	$(C' \in \overrightarrow{A'B'})$
{1}	$\overline{A'B'} \cong \overline{A'B'}$

Applying CongruentTransivity
where

`SegmentSelfCongruent:`

{-1}	$(\forall\, (s_1, s_2, s_3 :\ \text{Shape}) :$ $s_1 \cong s_2 \wedge s_2 \cong s_3 \Rightarrow s_1 \cong s_3)$
{-2}	$\overline{A'B'} \cong \overline{A'C'}$
{-3}	$(C' \in \overrightarrow{A'B'})$
{1}	$\overline{A'B'} \cong \overline{A'B'}$

Instantiating the top quantifier in -1 with the terms: $\overline{A'B'}$
, $\overline{A'C'}$
, $\overline{A'B'}$
,

`SegmentSelfCongruent:`

{-1}	$\overline{A'B'} \cong \overline{A'C'} \wedge$ $\overline{A'C'} \cong \overline{A'B'} \Rightarrow$ $\overline{A'B'} \cong \overline{A'B'}$
{-2}	$\overline{A'B'} \cong \overline{A'C'}$
{-3}	$(C' \in \overrightarrow{A'B'})$
{1}	$\overline{A'B'} \cong \overline{A'B'}$

Applying propositional simplification and decision procedures,

`SegmentSelfCongruent:`

{-1}	$\overline{A'B'} \cong \overline{A'C'}$
{-2}	$(C' \in \overrightarrow{A'B'})$
{1}	$\overline{A'C'} \cong \overline{A'B'}$
{2}	$\overline{A'B'} \cong \overline{A'B'}$

Applying CongruentSymmetry
where

`SegmentSelfCongruent:`

{-1}	$(\forall\, (s_1, s_2 :\ \text{Shape}) :\ s_1 \cong s_2 \Leftrightarrow s_2 \cong s_1)$
{-2}	$\overline{A'B'} \cong \overline{A'C'}$
{-3}	$(C' \in \overrightarrow{A'B'})$
{1}	$\overline{A'C'} \cong \overline{A'B'}$
{2}	$\overline{A'B'} \cong \overline{A'B'}$

Instantiating the top quantifier in -1 with the terms: $\overline{A'C'}$
, $\overline{A'B'}$
,

`SegmentSelfCongruent:`

{-1}	$\overline{A'C'} \cong \overline{A'B'} \Leftrightarrow$ $\overline{A'B'} \cong \overline{A'C'}$
{-2}	$\overline{A'B'} \cong \overline{A'C'}$
{-3}	$(C' \in \overrightarrow{A'B'})$
{1}	$\overline{A'C'} \cong \overline{A'B'}$
{2}	$\overline{A'B'} \cong \overline{A'B'}$

Applying propositional simplification and decision procedures,

This completes the proof of `SegmentSelfCongruent`.

Q.E.D.

B.3 Segment as intersection of rays

Verbose proof for `SegmentFromRay`.

`SegmentFromRay:`

$$\begin{array}{|ll}
\hline
\{1\} & \forall\,(A, B:\ \text{Point}):\\
& \quad A \neq B \Rightarrow \overline{AB} = (\overrightarrow{AB} \cap \overrightarrow{BA})
\end{array}$$

Skolemizing,

`SegmentFromRay:`

$$\begin{array}{|ll}
\hline
\{1\} & A' \neq B' \Rightarrow\\
& \quad \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}$$

Adding extensionality axioms for type: Shape

,

`SegmentFromRay:`

$$\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(\text{f_31}:\ \text{Shape}, \text{g_32}:\ \text{Shape}):\\
& \quad (\forall\,(\text{x_33}:\ \text{Point}):\ \text{f_31}(\text{x_33}) = \text{g_32}(\text{x_33})) \supset\\
& \quad\ \text{f_31} = \text{g_32})\\
\hline
\{1\} & A' \neq B' \Rightarrow\\
& \quad \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}$$

Instantiating the top quantifier in -1 with the terms: $\overline{A'B'}$
, $(\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$
,

`SegmentFromRay:`

$$\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(\text{x_33}:\ \text{Point}):\\
& \quad \overline{A'B'}(\text{x_33}) =\\
& \quad\ (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(\text{x_33})) \supset\\
& \quad \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})\\
\hline
\{1\} & A' \neq B' \Rightarrow\\
& \quad \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}$$

Applying propositional simplification and decision procedures,

`SegmentFromRay:`

$$\begin{array}{|ll}
\hline
\{1\} & (\forall\,(\text{x_33}:\ \text{Point}):\\
& \quad \overline{A'B'}(\text{x_33}) =\\
& \quad\ (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(\text{x_33}))\\
\{2\} & A' = B'\\
\{3\} & \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}$$

For the top quantifier in 1, we introduce Skolem constants: x
,

`SegmentFromRay:`

{1}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{2}	$A' = B'$
{3}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Applying SegmentOrder
where

`SegmentFromRay:`

{-1}	$(\forall\ (A, B, C :\ \text{Point}) :$ $(C \in \overline{AB}) \wedge$ $C \neq A \wedge C \neq B \wedge A \neq B \Rightarrow$ $(\neg(B \in \overline{AC})) \wedge$ $(\neg(A \in \overline{BC})))$
{1}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{2}	$A' = B'$
{3}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Instantiating the top quantifier in -1 with the terms: A!1
, B!1
, x
,

`SegmentFromRay:`

{-1}	$(x \in \overline{A'B'}) \wedge$ $x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow$ $(\neg(B' \in \overline{A'x})) \wedge$ $(\neg(A' \in \overline{B'x}))$
{1}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{2}	$A' = B'$
{3}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Applying RayDef
where

`SegmentFromRay:`

{-1}	$(\forall\ (A, B, C :\ \text{Point}) :$
	$(C \in \overrightarrow{AB}) \Leftrightarrow$
	$(C \in \overline{AB}) \vee (B \in \overline{AC}))$
{-2}	$(x \in \overline{A'B'}) \wedge$
	$x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow$
	$(\neg(B' \in \overline{A'x})) \wedge$
	$(\neg(A' \in \overline{B'x}))$
{1}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{2}	$A' = B'$
{3}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Instantiating (with copying) the top quantifier in -1 with the terms: A!1
, B!1
, x
,

`SegmentFromRay:`

{-1}	$(\forall\ (A, B, C :\ \text{Point}) :$
	$(C \in \overrightarrow{AB}) \Leftrightarrow$
	$(C \in \overline{AB}) \vee (B \in \overline{AC}))$
{-2}	$(x \in \overrightarrow{A'B'}) \Leftrightarrow$
	$(x \in \overline{A'B'}) \vee (B' \in \overline{A'x})$
{-3}	$(x \in \overline{A'B'}) \wedge$
	$x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow$
	$(\neg(B' \in \overline{A'x})) \wedge$
	$(\neg(A' \in \overline{B'x}))$
{1}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{2}	$A' = B'$
{3}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Instantiating the top quantifier in -1 with the terms: B!1
, A!1
, x
,

`SegmentFromRay:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (x \in \overrightarrow{B'A'}) \Leftrightarrow \\
 & \quad (x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \\
\{\text{-2}\} & (x \in \overrightarrow{A'B'}) \Leftrightarrow \\
 & \quad (x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \\
\{\text{-3}\} & (x \in \overline{A'B'}) \wedge \\
 & \quad x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow \\
 & \quad (\neg(B' \in \overline{A'x})) \wedge \\
 & \quad (\neg(A' \in \overline{B'x})) \\
\hline
\{1\} & \overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x) \\
\{2\} & A' = B' \\
\{3\} & \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}
$$

Applying SegmentSymmetry
where

`SegmentFromRay:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (\forall\ (A, B:\ \text{Point}):\ \overline{AB} = \overline{BA}) \\
\{\text{-2}\} & (x \in \overrightarrow{B'A'}) \Leftrightarrow \\
 & \quad (x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \\
\{\text{-3}\} & (x \in \overrightarrow{A'B'}) \Leftrightarrow \\
 & \quad (x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \\
\{\text{-4}\} & (x \in \overline{A'B'}) \wedge \\
 & \quad x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow \\
 & \quad (\neg(B' \in \overline{A'x})) \wedge \\
 & \quad (\neg(A' \in \overline{B'x})) \\
\hline
\{1\} & \overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x) \\
\{2\} & A' = B' \\
\{3\} & \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}
$$

Instantiating the top quantifier in -1 with the terms: A!1
, B!1
,

`SegmentFromRay:`

$$
\begin{array}{|ll}
\{\text{-1}\} & \overline{A'B'} = \overline{B'A'} \\
\{\text{-2}\} & (x \in \overrightarrow{B'A'}) \Leftrightarrow \\
 & \quad (x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \\
\{\text{-3}\} & (x \in \overrightarrow{A'B'}) \Leftrightarrow \\
 & \quad (x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \\
\{\text{-4}\} & (x \in \overline{A'B'}) \wedge \\
 & \quad x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow \\
 & \quad (\neg(B' \in \overline{A'x})) \wedge \\
 & \quad (\neg(A' \in \overline{B'x})) \\
\hline
\{1\} & \overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x) \\
\{2\} & A' = B' \\
\{3\} & \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}
$$

Applying disjunctive simplification to flatten sequent,

`SegmentFromRay:`

$$
\begin{array}{|ll}
\{\text{-1}\} & \overline{A'B'} = \overline{B'A'} \\
\{\text{-2}\} & (x \in \overrightarrow{B'A'}) \supset \\
 & \quad (x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \\
\{\text{-3}\} & (x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \supset \\
 & \quad (x \in \overrightarrow{B'A'}) \\
\{\text{-4}\} & (x \in \overrightarrow{A'B'}) \supset \\
 & \quad (x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \\
\{\text{-5}\} & (x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \supset \\
 & \quad (x \in \overrightarrow{A'B'}) \\
\{\text{-6}\} & (x \in \overline{A'B'}) \wedge \\
 & \quad x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow \\
 & \quad (\neg(B' \in \overline{A'x})) \wedge \\
 & \quad (\neg(A' \in \overline{B'x})) \\
\hline
\{1\} & \overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x) \\
\{2\} & A' = B' \\
\{3\} & \overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})
\end{array}
$$

Case splitting on $\overline{A'B'}(x)$

,

we get 2 subgoals:

`SegmentFromRay.1:`

{-1}	$\overline{A'B'}(x)$
{-2}	$\overline{A'B'} = \overline{B'A'}$
{-3}	$(x \in \overrightarrow{B'A'}) \supset$
	$(x \in \overline{B'A'}) \vee (A' \in \overline{B'x})$
{-4}	$(x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \supset$
	$(x \in \overrightarrow{B'A'})$
{-5}	$(x \in \overrightarrow{A'B'}) \supset$
	$(x \in \overline{A'B'}) \vee (B' \in \overline{A'x})$
{-6}	$(x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \supset$
	$(x \in \overrightarrow{A'B'})$
{-7}	$(x \in \overline{A'B'}) \wedge$
	$x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow$
	$(\neg(B' \in \overline{A'x})) \wedge$
	$(\neg(A' \in \overline{B'x}))$
{1}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{2}	$A' = B'$
{3}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Trying repeated skolemization, instantiation, and if-lifting,

This completes the proof of `SegmentFromRay.1`.

`SegmentFromRay.2:`

{-1}	$\overline{A'B'} = \overline{B'A'}$
{-2}	$(x \in \overrightarrow{B'A'}) \supset$
	$(x \in \overline{B'A'}) \vee (A' \in \overline{B'x})$
{-3}	$(x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \supset$
	$(x \in \overrightarrow{B'A'})$
{-4}	$(x \in \overrightarrow{A'B'}) \supset$
	$(x \in \overline{A'B'}) \vee (B' \in \overline{A'x})$
{-5}	$(x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \supset$
	$(x \in \overrightarrow{A'B'})$
{-6}	$(x \in \overline{A'B'}) \wedge$
	$x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow$
	$(\neg(B' \in \overline{A'x})) \wedge$
	$(\neg(A' \in \overline{B'x}))$
{1}	$\overline{A'B'}(x)$
{2}	$\overline{A'B'}(x) = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})(x)$
{3}	$A' = B'$
{4}	$\overline{A'B'} = (\overrightarrow{A'B'} \cap \overrightarrow{B'A'})$

Expanding the definition of intersection

,

`SegmentFromRay.2:`

{-1}	$\overline{A'B'} = \overline{B'A'}$
{-2}	$(x \in \overrightarrow{B'A'}) \supset$ $(x \in \overline{B'A'}) \vee (A' \in \overline{B'x})$
{-3}	$(x \in \overline{B'A'}) \vee (A' \in \overline{B'x}) \supset$ $(x \in \overrightarrow{B'A'})$
{-4}	$(x \in \overrightarrow{A'B'}) \supset$ $(x \in \overline{A'B'}) \vee (B' \in \overline{A'x})$
{-5}	$(x \in \overline{A'B'}) \vee (B' \in \overline{A'x}) \supset$ $(x \in \overrightarrow{A'B'})$
{-6}	$(x \in \overline{A'B'}) \wedge$ $x \neq A' \wedge x \neq B' \wedge A' \neq B' \Rightarrow$ $(\neg(B' \in \overline{A'x})) \wedge$ $(\neg(A' \in \overline{B'x}))$
{1}	$\overline{A'B'}(x)$
{2}	$(\overline{A'B'}(x) =$ $(x \in \overrightarrow{A'B'}) \wedge (x \in \overrightarrow{B'A'}))$
{3}	$A' = B'$
{4}	$(\overline{A'B'} =$ $\{x : \text{Point} \mid$ $(x \in \overrightarrow{A'B'}) \wedge (x \in \overrightarrow{B'A'})\})$

Expanding the definition of member

,

`SegmentFromRay.2:`

{-1}	$\overline{A'B'} = \overline{B'A'}$
{-2}	$(\overrightarrow{B'A'}\ (x) \supset (\overline{B'A'}(x) \vee \overline{B'x}(A')))$
{-3}	$((\overline{B'A'}(x) \vee \overline{B'x}(A')) \supset \overrightarrow{B'A'}\ (x))$
{-4}	$(\overrightarrow{A'B'}\ (x) \supset (\overline{A'B'}(x) \vee \overline{A'x}(B')))$
{-5}	$((\overline{A'B'}(x) \vee \overline{A'x}(B')) \supset \overrightarrow{A'B'}\ (x))$
{-6}	$((\overline{A'B'}(x) \wedge x \neq A' \wedge x \neq B') \Rightarrow$ $((\neg\overline{A'x}(B')) \wedge (\neg\overline{B'x}(A'))))$
{1}	$\overline{A'B'}(x)$
{2}	$(\overline{A'B'}(x) = (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x)))$
{3}	$A' = B'$
{4}	$(\overline{A'B'} =$ $\{x : \text{Point} \mid (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))\})$

Replacing using formula -1,

`SegmentFromRay.2:`

$$
\begin{array}{ll}
\{\text{-1}\} & \overline{A'B'} = \overline{B'A'} \\
\{\text{-2}\} & (\overrightarrow{B'A'}\ (x) \supset (\overline{B'A'}(x) \vee \overline{B'x}(A'))) \\
\{\text{-3}\} & ((\overline{B'A'}(x) \vee \overline{B'x}(A')) \supset \overrightarrow{B'A'}\ (x)) \\
\{\text{-4}\} & (\overrightarrow{A'B'}\ (x) \supset (\overline{B'A'}(x) \vee \overline{A'x}(B'))) \\
\{\text{-5}\} & ((\overline{B'A'}(x) \vee \overline{A'x}(B')) \supset \overrightarrow{A'B'}\ (x)) \\
\{\text{-6}\} & ((\overline{B'A'}(x) \wedge x \neq A' \wedge x \neq B') \Rightarrow \\
 & \quad ((\neg\overline{A'x}(B')) \wedge (\neg\overline{B'x}(A')))) \\
\hline
\{1\} & \overline{B'A'}(x) \\
\{2\} & (\overline{B'A'}(x) = (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))) \\
\{3\} & A' = B' \\
\{4\} & (\overline{B'A'} = \\
 & \quad \{x : \text{Point} \mid (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))\})
\end{array}
$$

Replacing using formula 1,

`SegmentFromRay.2:`

$$
\begin{array}{ll}
\{\text{-1}\} & \overline{A'B'} = \overline{B'A'} \\
\{\text{-2}\} & (\overrightarrow{B'A'}\ (x) \supset (\text{FALSE} \vee \overline{B'x}(A'))) \\
\{\text{-3}\} & ((\text{FALSE} \vee \overline{B'x}(A')) \supset \overrightarrow{B'A'}\ (x)) \\
\{\text{-4}\} & (\overrightarrow{A'B'}\ (x) \supset (\text{FALSE} \vee \overline{A'x}(B'))) \\
\{\text{-5}\} & ((\text{FALSE} \vee \overline{A'x}(B')) \supset \overrightarrow{A'B'}\ (x)) \\
\{\text{-6}\} & ((\text{FALSE} \wedge x \neq A' \wedge x \neq B') \Rightarrow \\
 & \quad ((\neg\overline{A'x}(B')) \wedge (\neg\overline{B'x}(A')))) \\
\hline
\{1\} & \overline{B'A'}(x) \\
\{2\} & (\text{FALSE} = (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))) \\
\{3\} & A' = B' \\
\{4\} & (\overline{B'A'} = \\
 & \quad \{x : \text{Point} \mid (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))\})
\end{array}
$$

Applying propositional simplification and decision procedures,

`SegmentFromRay.2:`

$$
\begin{array}{ll}
\{\text{-1}\} & \overrightarrow{A'B'}\ (x) \\
\{\text{-2}\} & \overline{A'x}(B') \\
\{\text{-3}\} & \overrightarrow{B'A'}\ (x) \\
\{\text{-4}\} & \overline{B'x}(A') \\
\{\text{-5}\} & \overline{A'B'} = \overline{B'A'} \\
\hline
\{1\} & \overline{B'A'}(x) \\
\{2\} & A' = B' \\
\{3\} & (\overline{B'A'} = \\
 & \quad \{x : \text{Point} \mid (\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))\})
\end{array}
$$

Applying SegmentOrder
where

`SegmentFromRay.2:`

$$
\begin{array}{ll}
\{\text{-1}\} & (\forall\,(A,B,C:\ \text{Point}):\\
& \quad (C \in \overline{AB})\wedge\\
& \qquad C \neq A \wedge C \neq B \wedge A \neq B \Rightarrow\\
& \quad (\neg(B \in \overline{AC}))\wedge\\
& \quad (\neg(A \in \overline{BC})))\\
\{\text{-2}\} & \overrightarrow{A'B'}\,(x)\\
\{\text{-3}\} & \overline{A'x}(B')\\
\{\text{-4}\} & \overrightarrow{B'A'}\,(x)\\
\{\text{-5}\} & \overline{B'x}(A')\\
\{\text{-6}\} & \overline{A'B'} = \overline{B'A'}\\
\hline
\{1\} & \overline{B'A'}(x)\\
\{2\} & A' = B'\\
\{3\} & (\overline{B'A'} =\\
& \quad \{x:\ \text{Point} \mid\ (\overrightarrow{A'B'}\,(x)\wedge\ \overrightarrow{B'A'}\,(x))\})
\end{array}
$$

Instantiating the top quantifier in -1 with the terms: A!1
, x
, B!1
,

`SegmentFromRay.2:`

$$
\begin{array}{ll}
\{\text{-1}\} & (B' \in \overline{A'x})\wedge\\
& \quad B' \neq A' \wedge B' \neq x \wedge A' \neq x \Rightarrow\\
& \quad (\neg(x \in \overline{A'B'}))\wedge\\
& \quad (\neg(A' \in \overline{xB'}))\\
\{\text{-2}\} & \overrightarrow{A'B'}\,(x)\\
\{\text{-3}\} & \overline{A'x}(B')\\
\{\text{-4}\} & \overrightarrow{B'A'}\,(x)\\
\{\text{-5}\} & \overline{B'x}(A')\\
\{\text{-6}\} & \overline{A'B'} = \overline{B'A'}\\
\hline
\{1\} & \overline{B'A'}(x)\\
\{2\} & A' = B'\\
\{3\} & (\overline{B'A'} =\\
& \quad \{x:\ \text{Point} \mid\ (\overrightarrow{A'B'}\,(x)\wedge\ \overrightarrow{B'A'}\,(x))\})
\end{array}
$$

Trying repeated skolemization, instantiation, and if-lifting,

`SegmentFromRay.2:`

{-1}	$\overrightarrow{A'B'}\,(x)$
{-2}	$\overline{A'x}(B')$
{-3}	$\overrightarrow{B'A'}\,(x)$
{-4}	$\overline{B'x}(A')$
{-5}	$\overline{A'B'} = \overline{B'A'}$
{1}	$(x \in \overline{B'A'})$
{2}	$\overline{xB'}(A')$
{3}	$\overline{B'A'}(x)$
{4}	$A' = B'$
{5}	$(\overline{B'A'} =$
	$\{x:\ \text{Point} \mid\ (\overrightarrow{A'B'}\,(x) \wedge \overrightarrow{B'A'}\,(x))\})$

Applying SegmentSymmetry
where

`SegmentFromRay.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ \overline{AB} = \overline{BA})$
{-2}	$\overrightarrow{A'B'}\,(x)$
{-3}	$\overline{A'x}(B')$
{-4}	$\overrightarrow{B'A'}\,(x)$
{-5}	$\overline{B'x}(A')$
{-6}	$\overline{A'B'} = \overline{B'A'}$
{1}	$(x \in \overline{B'A'})$
{2}	$\overline{xB'}(A')$
{3}	$\overline{B'A'}(x)$
{4}	$A' = B'$
{5}	$(\overline{B'A'} =$
	$\{x:\ \text{Point} \mid\ (\overrightarrow{A'B'}\,(x) \wedge \overrightarrow{B'A'}\,(x))\})$

Instantiating the top quantifier in -1 with the terms: B!1
, x
,

`SegmentFromRay.2:`

{-1}	$\overline{B'x} = \overline{xB'}$
{-2}	$\overrightarrow{A'B'}\ (x)$
{-3}	$\overline{A'x}(B')$
{-4}	$\overrightarrow{B'A'}\ (x)$
{-5}	$\overline{B'x}(A')$
{-6}	$\overline{A'B'} = \overline{B'A'}$
{1}	$(x \in \overline{B'A'})$
{2}	$\overline{xB'}(A')$
{3}	$\overline{B'A'}(x)$
{4}	$A' = B'$
{5}	$(\overline{B'A'} =$
	$\{x :$ Point $\mid$ $(\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))\})$

Replacing using formula -1,

`SegmentFromRay.2:`

{-1}	$\overline{B'x} = \overline{xB'}$
{-2}	$\overrightarrow{A'B'}\ (x)$
{-3}	$\overline{A'x}(B')$
{-4}	$\overrightarrow{B'A'}\ (x)$
{-5}	$\overline{xB'}(A')$
{-6}	$\overline{A'B'} = \overline{B'A'}$
{1}	$(x \in \overline{B'A'})$
{2}	$\overline{xB'}(A')$
{3}	$\overline{B'A'}(x)$
{4}	$A' = B'$
{5}	$(\overline{B'A'} =$
	$\{x :$ Point $\mid$ $(\overrightarrow{A'B'}\ (x) \wedge \overrightarrow{B'A'}\ (x))\})$

which is trivially true.

This completes the proof of `SegmentFromRay.2`.

Q.E.D.

B.4 Dividing a Segment

Verbose proof for `SegmentDivision`.

`SegmentDivision:`

$$
\begin{array}{|ll}
\hline
\{1\} & \forall\,(A, C:\ \text{Point}):\\
& \quad A \neq C \Rightarrow\\
& \quad\ \exists(D:\ \text{Point}):\\
& \quad\quad D \neq A \wedge D \neq C \wedge (D \in \overline{AC})
\end{array}
$$

Skolemizing,

`SegmentDivision:`

$$
\begin{array}{|ll}
\hline
\{1\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\ D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Applying ThreePoints
where

`SegmentDivision:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(l:\ \text{Line}):\ \exists\,(C:\ \text{Point}):\ \neg(C \in l))\\
\hline
\{1\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\ D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Instantiating the top quantifier in -1 with the terms: $\overleftrightarrow{A'C'}$
,

`SegmentDivision:`

$$
\begin{array}{|ll}
\{\text{-1}\} & \exists\,(C:\ \text{Point}):\ \neg(C \in \overleftrightarrow{A'C'})\\
\hline
\{1\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\ D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

For the top quantifier in -1, we introduce Skolem constants: E
,

`SegmentDivision:`

$$
\begin{array}{|ll}
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\ D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Applying SegmentExtension
where

`SegmentDivision:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(A, B:\ \text{Point}):\\
& \quad \exists\,(C:\ \text{Point}):\\
& \quad\quad (B \in \overline{AC}) \wedge C \neq B \wedge C \neq A)\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Instantiating (with copying) the top quantifier in -1 with the terms: A!1
, E
,

`SegmentDivision:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(A, B:\ \text{Point}):\\
& \quad \exists\,(C:\ \text{Point}):\\
& \quad\quad (B \in \overline{AC}) \wedge C \neq B \wedge C \neq A)\\
\{\text{-2}\} & \exists\,(C:\ \text{Point}):\\
& \quad (E \in \overline{A'C}) \wedge C \neq E \wedge C \neq A'\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

For the top quantifier in -2, we introduce Skolem constants: F
,

`SegmentDivision:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (\forall\,(A, B:\ \text{Point}):\\
& \quad \exists\,(C:\ \text{Point}):\\
& \quad\quad (B \in \overline{AC}) \wedge C \neq B \wedge C \neq A)\\
\{\text{-2}\} & (E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Instantiating the top quantifier in -1 with the terms: F
, C!1
,

`SegmentDivision:`

$$
\begin{array}{|ll}
\{\text{-1}\} & \exists\,(C:\ \text{Point}):\\
& \quad (C' \in \overline{FC}) \wedge C \neq C' \wedge C \neq F\\
\{\text{-2}\} & (E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \quad \exists(D:\ \text{Point}):\\
& \quad\quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

For the top quantifier in -1, we introduce Skolem constants: G

,

`SegmentDivision:`

{-1}	$(C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F$
{-2}	$(E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'$
{1}	$(E \in \overleftrightarrow{A'C'})$
{2}	$A' \neq C' \Rightarrow$
	$\exists(D:\ \text{Point}):$
	$D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})$

Applying TriangleIntersection
where

`SegmentDivision:`

{-1}	$(\forall\ (A, B, C:\ \text{Point}, l:\ \text{Line}):$
	$(\exists\ (D:\ \text{Point}):$
	$(D \in (\overline{AB} \cap l))\wedge$
	$D \neq A \wedge D \neq B) \Rightarrow$
	$\exists(E:\ \text{Point}):$
	$(E \in$
	$((\overline{AC} \cup$
	$\overline{BC}) \cap$
	$l))\wedge$
	$E \neq A \wedge E \neq B)$
{-2}	$(C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F$
{-3}	$(E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'$
{1}	$(E \in \overleftrightarrow{A'C'})$
{2}	$A' \neq C' \Rightarrow$
	$\exists(D:\ \text{Point}):$
	$D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})$

Instantiating the top quantifier in -1 with the terms: A!1
, F
, C!1
, $\overleftrightarrow{EG}$
,

`SegmentDivision:`

$$
\begin{array}{ll}
\{\text{-1}\} & (\exists\,(D:\ \text{Point}):\\
& \quad (D \in (\overline{A'F} \cap \overleftrightarrow{EG}))\wedge\\
& \quad\ D \neq A' \wedge D \neq F) \Rightarrow\\
& \ \exists(E:\ \text{Point}):\\
& \quad (E \in\\
& \qquad ((\overline{A'C'} \cup\\
& \qquad\quad \overline{FC'}) \cap\\
& \qquad\ \overleftrightarrow{EG}))\wedge\\
& \quad\ E \neq A' \wedge E \neq F\\
\{\text{-2}\} & (C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F\\
\{\text{-3}\} & (E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \ \exists(D:\ \text{Point}):\\
& \quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Splitting conjunctions,

we get 2 subgoals:

`SegmentDivision.1:`

$$
\begin{array}{ll}
\{\text{-1}\} & \exists\,(E:\ \text{Point}):\\
& \ (E \in\\
& \qquad ((\overline{A'C'} \cup\\
& \qquad\quad \overline{FC'}) \cap\\
& \qquad\ \overleftrightarrow{EG}))\wedge\\
& \quad\ E \neq A' \wedge E \neq F\\
\{\text{-2}\} & (C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F\\
\{\text{-3}\} & (E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \ \exists(D:\ \text{Point}):\\
& \quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

For the top quantifier in -1, we introduce Skolem constants: E!1

,

`SegmentDivision.1:`

$$
\begin{array}{ll}
\{\text{-1}\} & (E' \in\\
& \quad ((\overline{A'C'} \cup \overline{FC'}) \cap\\
& \quad\ \overleftrightarrow{EG}))\wedge\\
& \ E' \neq A' \wedge E' \neq F\\
\{\text{-2}\} & (C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F\\
\{\text{-3}\} & (E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'\\
\hline
\{1\} & (E \in \overleftrightarrow{A'C'})\\
\{2\} & A' \neq C' \Rightarrow\\
& \ \exists(D:\ \text{Point}):\\
& \quad D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})
\end{array}
$$

Applying disjunctive simplification to flatten sequent,

`SegmentDivision.1:`

{-1}	$(E' \in ((\overline{A'C'} \cup \overline{FC'}) \cap \overleftrightarrow{EG}))$
{-2}	$(C' \in \overline{FG})$
{-3}	$(E \in \overline{A'F})$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$(E \in \overleftrightarrow{A'C'})$
{8}	$A' = C'$
{9}	$\exists\, (D :\ \text{Point}) : D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})$

Instantiating the top quantifier in 9 with the terms: E!1

,

`SegmentDivision.1:`

{-1}	$(E' \in ((\overline{A'C'} \cup \overline{FC'}) \cap \overleftrightarrow{EG}))$
{-2}	$(C' \in \overline{FG})$
{-3}	$(E \in \overline{A'F})$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$(E \in \overleftrightarrow{A'C'})$
{8}	$A' = C'$
{9}	$E' \neq A' \wedge E' \neq C' \wedge (E' \in \overline{A'C'})$

Expanding the definition of intersection

,

`SegmentDivision.1:`

{-1}	$(E' \in$ $\{x:$ Point $\mid$ $(x \in (\overline{A'C'} \cup \overline{FC'}))\wedge$ $(x \in \overleftrightarrow{EG})\})$
{-2}	$(C' \in \overline{FG})$
{-3}	$(E \in \overline{A'F})$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$(E \in \overleftrightarrow{A'C'})$
{8}	$A' = C'$
{9}	$E' \neq A' \wedge E' \neq C' \wedge (E' \in \overline{A'C'})$

Expanding the definition of union

,

`SegmentDivision.1:`

{-1}	$(E' \in$ $\{x:$ Point $\mid$ $((x \in$ $\{x:$ Point $\mid$ $(x \in \overline{A'C'})\vee$ $(x \in \overline{FC'})\})\wedge$ $(x \in \overleftrightarrow{EG}))\})$
{-2}	$(C' \in \overline{FG})$
{-3}	$(E \in \overline{A'F})$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$(E \in \overleftrightarrow{A'C'})$
{8}	$A' = C'$
{9}	$E' \neq A' \wedge E' \neq C' \wedge (E' \in \overline{A'C'})$

Expanding the definition of member

,

`SegmentDivision.1:`

{-1}	$((\overline{A'C'}(E') \vee \overline{FC'}(E')) \wedge \overleftrightarrow{EG}\,(E'))$
{-2}	$\overline{FG}(C')$
{-3}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$
{9}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Applying disjunctive simplification to flatten sequent,

`SegmentDivision.1:`

{-1}	$(\overline{A'C'}(E') \vee \overline{FC'}(E'))$
{-2}	$\overleftrightarrow{EG}\,(E')$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$
{9}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Splitting conjunctions,

we get 2 subgoals:

`SegmentDivision.1.1:`

{-1}	$\overline{A'C'}(E')$
{-2}	$\overleftrightarrow{EG}\,(E')$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$
{9}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Replacing using formula -1,

`SegmentDivision.1.1:`

{-1}	$\overline{A'C'}(E')$
{-2}	$\overleftrightarrow{EG}\,(E')$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$
{9}	$(E' \neq A' \wedge (E' \neq C' \wedge \mathrm{TRUE}))$

Applying propositional simplification and decision procedures,

`SegmentDivision.1.1:`

{-1}	$E' = C'$
{-2}	$\overline{A'C'}(E')$
{-3}	$\overleftrightarrow{EG}\,(E')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$

Replacing using formula -1,

`SegmentDivision.1.1:`

{-1}	$E' = C'$
{-2}	$\overline{A'C'}(C')$
{-3}	$\overleftrightarrow{EG}\,(C')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$

Hiding formulas: -1, -2,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{EG}\,(C')$
{-2}	$\overline{FG}(C')$
{-3}	$\overline{A'F}(E)$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$

Applying SegmentOnLine
where

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (\overline{AB} \subseteq \overleftrightarrow{AB}))$
{-2}	$\overleftrightarrow{EG}\,(C')$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(E)$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, G
,

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (\overline{AB} \subseteq \overleftrightarrow{AB}))$
{-2}	$(\overline{FG} \subseteq \overleftrightarrow{FG})$
{-3}	$\overleftrightarrow{EG}\,(C')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: A!1
, F
,

`SegmentDivision.1.1:`

{-1}	$(\overline{A'F} \subseteq \overleftrightarrow{A'F})$
{-2}	$(\overline{FG} \subseteq \overleftrightarrow{FG})$
{-3}	$\overleftrightarrow{EG}\ (C')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Trying repeated skolemization, instantiation, and if-lifting,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{A'F}\ (E)$
{-2}	$\overleftrightarrow{FG}\ (C')$
{-3}	$\overleftrightarrow{EG}\ (C')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Hiding formulas: -4, -5,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{A'F}\ (E)$
{-2}	$\overleftrightarrow{FG}\ (C')$
{-3}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Applying OnlyLine
where

`SegmentDivision.1.1:`

{-1}	$(\forall\ (A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$\overleftrightarrow{A'F}\ (E)$
{-3}	$\overleftrightarrow{FG}\ (C')$
{-4}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: G
, C!1
, $\overleftrightarrow{FG}$
,

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(G \in \overleftrightarrow{FG}) \wedge (C' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{GC'} \vee G = C'$
{-3}	$\overleftrightarrow{A'F}\ (E)$
{-4}	$\overleftrightarrow{FG}\ (C')$
{-5}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: G
, C!1
, $\overleftrightarrow{EG}$
,

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(G \in \overleftrightarrow{EG}) \wedge (C' \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{GC'} \vee G = C'$
{-3}	$(G \in \overleftrightarrow{FG}) \wedge (C' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{GC'} \vee G = C'$
{-4}	$\overleftrightarrow{A'F}\ (E)$
{-5}	$\overleftrightarrow{FG}\ (C')$
{-6}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, E
, $\overleftrightarrow{A'F}$
,

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-3}	$(G \in \overleftrightarrow{EG}) \wedge (C' \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{GC'} \vee G = C'$
{-4}	$(G \in \overleftrightarrow{FG}) \wedge (C' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{GC'} \vee G = C'$
{-5}	$\overleftrightarrow{A'F}\ (E)$
{-6}	$\overleftrightarrow{FG}\ (C')$
{-7}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, E
, $\overleftrightarrow{GC'}$
,

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(F \in \overleftrightarrow{GC'}) \wedge (E \in \overleftrightarrow{GC'}) \Rightarrow$ $\overleftrightarrow{GC'} = \overleftrightarrow{FE} \vee F = E$
{-3}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-4}	$(G \in \overleftrightarrow{EG}) \wedge (C' \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{GC'} \vee G = C'$
{-5}	$(G \in \overleftrightarrow{FG}) \wedge (C' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{GC'} \vee G = C'$
{-6}	$\overleftrightarrow{A'F}\ (E)$
{-7}	$\overleftrightarrow{FG}\ (C')$
{-8}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: A!1
, C!1
, $\overleftrightarrow{A'F}$
,

`SegmentDivision.1.1:`

{-1}	$(A' \in \overleftrightarrow{A'F}) \wedge (C' \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{A'C'} \vee A' = C'$
{-2}	$(F \in \overleftrightarrow{GC'}) \wedge (E \in \overleftrightarrow{GC'}) \Rightarrow$ $\overleftrightarrow{GC'} = \overleftrightarrow{FE} \vee F = E$
{-3}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-4}	$(G \in \overleftrightarrow{EG}) \wedge (C' \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{GC'} \vee G = C'$
{-5}	$(G \in \overleftrightarrow{FG}) \wedge (C' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{GC'} \vee G = C'$
{-6}	$\overleftrightarrow{A'F}\ (E)$
{-7}	$\overleftrightarrow{FG}\ (C')$
{-8}	$\overleftrightarrow{EG}\ (C')$
{1}	$C' = A'$
{2}	$C' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.1.1:`

{-1}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow \overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-2}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-3}	$(\overleftrightarrow{EG}\ (G) \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-4}	$(\overleftrightarrow{FG}\ (G) \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-5}	$\overleftrightarrow{A'F}\ (E)$
{-6}	$\overleftrightarrow{FG}\ (C')$
{-7}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying LineExists
where

`SegmentDivision.1.1:`

{-1}	$(\forall\ (A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB}))$
{-2}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow \overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-3}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-4}	$(\overleftrightarrow{EG}\ (G) \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-5}	$(\overleftrightarrow{FG}\ (G) \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-6}	$\overleftrightarrow{A'F}\ (E)$
{-7}	$\overleftrightarrow{FG}\ (C')$
{-8}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: E
, G
,

`SegmentDivision.1.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB}))$
{-2}	$(E \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG})$
{-3}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow$ $\overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-4}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-5}	$(\overleftrightarrow{EG}\ (G) \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-6}	$(\overleftrightarrow{FG}\ (G) \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-7}	$\overleftrightarrow{A'F}\ (E)$
{-8}	$\overleftrightarrow{FG}\ (C')$
{-9}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: F
, G
,

`SegmentDivision.1.1:`

{-1}	$(F \in \overleftrightarrow{FG}) \wedge (G \in \overleftrightarrow{FG})$
{-2}	$(E \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG})$
{-3}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow$ $\overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-4}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-5}	$(\overleftrightarrow{EG}\ (G) \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-6}	$(\overleftrightarrow{FG}\ (G) \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-7}	$\overleftrightarrow{A'F}\ (E)$
{-8}	$\overleftrightarrow{FG}\ (C')$
{-9}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.1.1:`

{-1}	$(\overleftrightarrow{FG}\,(F) \wedge \overleftrightarrow{FG}\,(G))$
{-2}	$(\overleftrightarrow{EG}\,(E) \wedge \overleftrightarrow{EG}\,(G))$
{-3}	$((\overleftrightarrow{GC'}\,(F) \wedge \overleftrightarrow{GC'}\,(E)) \Rightarrow$
	$\overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-4}	$(\overleftrightarrow{A'F}\,(F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-5}	$(\overleftrightarrow{EG}\,(G) \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-6}	$(\overleftrightarrow{FG}\,(G) \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-7}	$\overleftrightarrow{A'F}\,(E)$
{-8}	$\overleftrightarrow{FG}\,(C')$
{-9}	$\overleftrightarrow{EG}\,(C')$
{1}	$(\overleftrightarrow{A'F}\,(A') \wedge \overleftrightarrow{A'F}\,(C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Applying disjunctive simplification to flatten sequent,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{FG}\ (F)$
{-2}	$\overleftrightarrow{FG}\ (G)$
{-3}	$\overleftrightarrow{EG}\ (E)$
{-4}	$\overleftrightarrow{EG}\ (G)$
{-5}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow \overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-6}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-7}	$(\overleftrightarrow{EG}\ (G) \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-8}	$(\overleftrightarrow{FG}\ (G) \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-9}	$\overleftrightarrow{A'F}\ (E)$
{-10}	$\overleftrightarrow{FG}\ (C')$
{-11}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Repeatedly applying the replace rule,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{FG}\ (F)$
{-2}	$\overleftrightarrow{FG}\ (G)$
{-3}	$\overleftrightarrow{EG}\ (E)$
{-4}	$\overleftrightarrow{EG}\ (G)$
{-5}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow \overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-6}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-7}	$(\text{TRUE} \Rightarrow \overleftrightarrow{EG} = \overleftrightarrow{GC'})$
{-8}	$(\text{TRUE} \Rightarrow \overleftrightarrow{FG} = \overleftrightarrow{GC'})$
{-9}	$\overleftrightarrow{A'F}\ (E)$
{-10}	$\overleftrightarrow{FG}\ (C')$
{-11}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Simplifying with decision procedures,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{FG}\ (F)$
{-2}	$\overleftrightarrow{FG}\ (G)$
{-3}	$\overleftrightarrow{EG}\ (E)$
{-4}	$\overleftrightarrow{EG}\ (G)$
{-5}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow \overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-6}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-7}	$\overleftrightarrow{EG} = \overleftrightarrow{GC'}$
{-8}	$\overleftrightarrow{FG} = \overleftrightarrow{GC'}$
{-9}	$\overleftrightarrow{A'F}\ (E)$
{-10}	$\overleftrightarrow{FG}\ (C')$
{-11}	$\overleftrightarrow{EG}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Repeatedly applying the replace rule,

`SegmentDivision.1.1:`

{-1}	$\overleftrightarrow{GC'}\ (F)$
{-2}	$\overleftrightarrow{GC'}\ (G)$
{-3}	$\overleftrightarrow{GC'}\ (E)$
{-4}	$\overleftrightarrow{GC'}\ (G)$
{-5}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow \overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-6}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-7}	$\overleftrightarrow{EG} = \overleftrightarrow{GC'}$
{-8}	$\overleftrightarrow{FG} = \overleftrightarrow{GC'}$
{-9}	$\overleftrightarrow{A'F}\ (E)$
{-10}	$\overleftrightarrow{GC'}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying LineExists
where A
gets A!1
, B
gets F
,

`SegmentDivision.1.1:`

{-1}	$(A' \in \overleftrightarrow{A'F}) \wedge (F \in \overleftrightarrow{A'F})$
{-2}	$\overleftrightarrow{GC'}\ (F)$
{-3}	$\overleftrightarrow{GC'}\ (G)$
{-4}	$\overleftrightarrow{GC'}\ (E)$
{-5}	$\overleftrightarrow{GC'}\ (G)$
{-6}	$((\overleftrightarrow{GC'}\ (F) \wedge \overleftrightarrow{GC'}\ (E)) \Rightarrow$ $\overleftrightarrow{GC'} = \overleftrightarrow{FE})$
{-7}	$(\overleftrightarrow{A'F}\ (F) \Rightarrow \overleftrightarrow{A'F} = \overleftrightarrow{FE})$
{-8}	$\overleftrightarrow{EG} = \overleftrightarrow{GC'}$
{-9}	$\overleftrightarrow{FG} = \overleftrightarrow{GC'}$
{-10}	$\overleftrightarrow{A'F}\ (E)$
{-11}	$\overleftrightarrow{GC'}\ (C')$
{1}	$(\overleftrightarrow{A'F}\ (A') \wedge \overleftrightarrow{A'F}\ (C'))$
{2}	$C' = A'$
{3}	$C' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Trying repeated skolemization, instantiation, and if-lifting,

This completes the proof of `SegmentDivision.1.1`.

`SegmentDivision.1.2:`

{-1}	$\overline{FC'}(E')$
{-2}	$\overleftrightarrow{EG}\ (E')$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$
{9}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Case splitting on $E' = G$

,

we get 2 subgoals:

`SegmentDivision.1.2.1:`

{-1}	$E' = G$
{-2}	$\overline{FC'}(E')$
{-3}	$\overleftrightarrow{EG}\,(E')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$E' = A'$
{2}	$E' = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$
{9}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Replacing using formula -1,

`SegmentDivision.1.2.1:`

{-1}	$E' = G$
{-2}	$\overline{FC'}(G)$
{-3}	$\overleftrightarrow{EG}\,(G)$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$G = A'$
{2}	$G = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\,(E)$
{8}	$A' = C'$
{9}	$(G \neq A' \wedge (G \neq C' \wedge \overline{A'C'}(G)))$

Applying SegmentOrder
where

`SegmentDivision.1.2.1:`

{-1}	$(\forall\ (A, B, C:\ \text{Point}):$
	$(C \in \overline{AB}) \wedge$
	$C \neq A \wedge C \neq B \wedge A \neq B \Rightarrow$
	$(\neg(B \in \overline{AC})) \wedge$
	$(\neg(A \in \overline{BC})))$
{-2}	$E' = G$
{-3}	$\overline{FC'}(G)$
{-4}	$\overleftrightarrow{EG}\ (G)$
{-5}	$\overline{FG}(C')$
{-6}	$\overline{A'F}(E)$
{1}	$G = A'$
{2}	$G = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$
{9}	$(G \neq A' \wedge (G \neq C' \wedge \overline{A'C'}(G)))$

Instantiating the top quantifier in -1 with the terms: F
, C!1
, G
,

`SegmentDivision.1.2.1:`

{-1}	$(G \in \overline{FC'}) \wedge$
	$G \neq F \wedge G \neq C' \wedge F \neq C' \Rightarrow$
	$(\neg(C' \in \overline{FG})) \wedge$
	$(\neg(F \in \overline{C'G}))$
{-2}	$E' = G$
{-3}	$\overline{FC'}(G)$
{-4}	$\overleftrightarrow{EG}\ (G)$
{-5}	$\overline{FG}(C')$
{-6}	$\overline{A'F}(E)$
{1}	$G = A'$
{2}	$G = F$
{3}	$G = C'$
{4}	$G = F$
{5}	$F = E$
{6}	$F = A'$
{7}	$\overleftrightarrow{A'C'}\ (E)$
{8}	$A' = C'$
{9}	$(G \neq A' \wedge (G \neq C' \wedge \overline{A'C'}(G)))$

Trying repeated skolemization, instantiation, and if-lifting,

`SegmentDivision.1.2.1:`

{-1}	$F = C'$
{-2}	$E' = G$
{-3}	$\overline{C'C'}(G)$
{-4}	$\overleftrightarrow{EG}\,(G)$
{-5}	$\overline{C'G}(C')$
{-6}	$\overline{A'C'}(E)$
{1}	$G = A'$
{2}	$G = C'$
{3}	$C' = E$
{4}	$C' = A'$
{5}	$\overleftrightarrow{A'C'}\,(E)$
{6}	$A' = C'$
{7}	$\overline{A'C'}(G)$

Applying SegmentOnLine
where

`SegmentDivision.1.2.1:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (\overline{AB} \subseteq \overleftrightarrow{AB}))$
{-2}	$F = C'$
{-3}	$E' = G$
{-4}	$\overline{C'C'}(G)$
{-5}	$\overleftrightarrow{EG}\,(G)$
{-6}	$\overline{C'G}(C')$
{-7}	$\overline{A'C'}(E)$
{1}	$G = A'$
{2}	$G = C'$
{3}	$C' = E$
{4}	$C' = A'$
{5}	$\overleftrightarrow{A'C'}\,(E)$
{6}	$A' = C'$
{7}	$\overline{A'C'}(G)$

Instantiating the top quantifier in -1 with the terms: A!1
, C!1
,

`SegmentDivision.1.2.1:`

{-1}	$(\overline{A'C'} \subseteq \overleftrightarrow{A'C'})$
{-2}	$F = C'$
{-3}	$E' = G$
{-4}	$\overline{C'C'}(G)$
{-5}	$\overleftrightarrow{EG}\,(G)$
{-6}	$\overline{C'G}(C')$
{-7}	$\overline{A'C'}(E)$
{1}	$G = A'$
{2}	$G = C'$
{3}	$C' = E$
{4}	$C' = A'$
{5}	$\overleftrightarrow{A'C'}\,(E)$
{6}	$A' = C'$
{7}	$\overline{A'C'}(G)$

Trying repeated skolemization, instantiation, and if-lifting,

This completes the proof of `SegmentDivision.1.2.1`.

`SegmentDivision.1.2.2:`

{-1}	$\overline{FC'}(E')$
{-2}	$\overleftrightarrow{EG}\,(E')$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$
{10}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Applying SegmentOnLine
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (\overline{AB} \subseteq \overleftrightarrow{AB}))$
{-2}	$\overline{FC'}(E')$
{-3}	$\overleftrightarrow{EG}\,(E')$
{-4}	$\overline{FG}(C')$
{-5}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$
{10}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, C!1
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (\overline{AB} \subseteq \overleftrightarrow{AB}))$
{-2}	$(\overline{FC'} \subseteq \overleftrightarrow{FC'})$
{-3}	$\overline{FC'}(E')$
{-4}	$\overleftrightarrow{EG}\,(E')$
{-5}	$\overline{FG}(C')$
{-6}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$
{10}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, G
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (\overline{AB} \subseteq \overleftrightarrow{AB}))$
{-2}	$(\overline{FG} \subseteq \overleftrightarrow{FG})$
{-3}	$(\overline{FC'} \subseteq \overleftrightarrow{FC'})$
{-4}	$\overline{FC'}(E')$
{-5}	$\overleftrightarrow{EG}\,(E')$
{-6}	$\overline{FG}(C')$
{-7}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$
{10}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Instantiating the top quantifier in -1 with the terms: A!1
, F
,

`SegmentDivision.1.2.2:`

{-1}	$(\overline{A'F} \subseteq \overleftrightarrow{A'F})$
{-2}	$(\overline{FG} \subseteq \overleftrightarrow{FG})$
{-3}	$(\overline{FC'} \subseteq \overleftrightarrow{FC'})$
{-4}	$\overline{FC'}(E')$
{-5}	$\overleftrightarrow{EG}\,(E')$
{-6}	$\overline{FG}(C')$
{-7}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$
{10}	$(E' \neq A' \wedge (E' \neq C' \wedge \overline{A'C'}(E')))$

Hiding formulas: 10,

`SegmentDivision.1.2.2:`

{-1}	$(\overline{A'F} \subseteq \overleftrightarrow{A'F})$
{-2}	$(\overline{FG} \subseteq \overleftrightarrow{FG})$
{-3}	$(\overline{FC'} \subseteq \overleftrightarrow{FC'})$
{-4}	$\overline{FC'}(E')$
{-5}	$\overleftrightarrow{EG}\,(E')$
{-6}	$\overline{FG}(C')$
{-7}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Trying repeated skolemization, instantiation, and if-lifting,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{A'F}\,(E)$
{-2}	$\overleftrightarrow{FG}\,(C')$
{-3}	$\overleftrightarrow{FC'}\,(E')$
{-4}	$\overline{FC'}(E')$
{-5}	$\overleftrightarrow{EG}\,(E')$
{-6}	$\overline{FG}(C')$
{-7}	$\overline{A'F}(E)$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Hiding formulas: -4, -6, -7,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{A'F}\,(E)$
{-2}	$\overleftrightarrow{FG}\,(C')$
{-3}	$\overleftrightarrow{FC'}\,(E')$
{-4}	$\overleftrightarrow{EG}\,(E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Applying LineExists
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB}))$
{-2}	$\overleftrightarrow{A'F}\,(E)$
{-3}	$\overleftrightarrow{FG}\,(C')$
{-4}	$\overleftrightarrow{FC'}\,(E')$
{-5}	$\overleftrightarrow{EG}\,(E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: A!1
, F
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB}))$
{-2}	$(A' \in \overleftrightarrow{A'F}) \wedge (F \in \overleftrightarrow{A'F})$
{-3}	$\overleftrightarrow{A'F}\,(E)$
{-4}	$\overleftrightarrow{FG}\,(C')$
{-5}	$\overleftrightarrow{FC'}\,(E')$
{-6}	$\overleftrightarrow{EG}\,(E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, G
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB}))$
{-2}	$(F \in \overleftrightarrow{FG}) \wedge (G \in \overleftrightarrow{FG})$
{-3}	$(A' \in \overleftrightarrow{A'F}) \wedge (F \in \overleftrightarrow{A'F})$
{-4}	$\overleftrightarrow{A'F}\,(E)$
{-5}	$\overleftrightarrow{FG}\,(C')$
{-6}	$\overleftrightarrow{FC'}\,(E')$
{-7}	$\overleftrightarrow{EG}\,(E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, C!1
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ (A \in \overleftrightarrow{AB}) \wedge (B \in \overleftrightarrow{AB}))$
{-2}	$(F \in \overleftrightarrow{FC'}) \wedge (C' \in \overleftrightarrow{FC'})$
{-3}	$(F \in \overleftrightarrow{FG}) \wedge (G \in \overleftrightarrow{FG})$
{-4}	$(A' \in \overleftrightarrow{A'F}) \wedge (F \in \overleftrightarrow{A'F})$
{-5}	$\overleftrightarrow{A'F}\ (E)$
{-6}	$\overleftrightarrow{FG}\ (C')$
{-7}	$\overleftrightarrow{FC'}\ (E')$
{-8}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: E
, G
,

`SegmentDivision.1.2.2:`

{-1}	$(E \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG})$
{-2}	$(F \in \overleftrightarrow{FC'}) \wedge (C' \in \overleftrightarrow{FC'})$
{-3}	$(F \in \overleftrightarrow{FG}) \wedge (G \in \overleftrightarrow{FG})$
{-4}	$(A' \in \overleftrightarrow{A'F}) \wedge (F \in \overleftrightarrow{A'F})$
{-5}	$\overleftrightarrow{A'F}\ (E)$
{-6}	$\overleftrightarrow{FG}\ (C')$
{-7}	$\overleftrightarrow{FC'}\ (E')$
{-8}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Trying repeated skolemization, instantiation, and if-lifting,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{EG}\ (E)$
{-2}	$\overleftrightarrow{EG}\ (G)$
{-3}	$\overleftrightarrow{FC'}\ (F)$
{-4}	$\overleftrightarrow{FC'}\ (C')$
{-5}	$\overleftrightarrow{FG}\ (F)$
{-6}	$\overleftrightarrow{FG}\ (G)$
{-7}	$\overleftrightarrow{A'F}\ (A')$
{-8}	$\overleftrightarrow{A'F}\ (F)$
{-9}	$\overleftrightarrow{A'F}\ (E)$
{-10}	$\overleftrightarrow{FG}\ (C')$
{-11}	$\overleftrightarrow{FC'}\ (E')$
{-12}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying OnlyLine
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$\overleftrightarrow{EG}\ (E)$
{-3}	$\overleftrightarrow{EG}\ (G)$
{-4}	$\overleftrightarrow{FC'}\ (F)$
{-5}	$\overleftrightarrow{FC'}\ (C')$
{-6}	$\overleftrightarrow{FG}\ (F)$
{-7}	$\overleftrightarrow{FG}\ (G)$
{-8}	$\overleftrightarrow{A'F}\ (A')$
{-9}	$\overleftrightarrow{A'F}\ (F)$
{-10}	$\overleftrightarrow{A'F}\ (E)$
{-11}	$\overleftrightarrow{FG}\ (C')$
{-12}	$\overleftrightarrow{FC'}\ (E')$
{-13}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, E
, $\overleftrightarrow{A'F}$
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-3}	$\overleftrightarrow{EG}\ (E)$
{-4}	$\overleftrightarrow{EG}\ (G)$
{-5}	$\overleftrightarrow{FC'}\ (F)$
{-6}	$\overleftrightarrow{FC'}\ (C')$
{-7}	$\overleftrightarrow{FG}\ (F)$
{-8}	$\overleftrightarrow{FG}\ (G)$
{-9}	$\overleftrightarrow{A'F}\ (A')$
{-10}	$\overleftrightarrow{A'F}\ (F)$
{-11}	$\overleftrightarrow{A'F}\ (E)$
{-12}	$\overleftrightarrow{FG}\ (C')$
{-13}	$\overleftrightarrow{FC'}\ (E')$
{-14}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: E!1
, G
, $\overleftrightarrow{EG}$
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(E' \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{E'G} \vee E' = G$
{-3}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-4}	$\overleftrightarrow{EG}\ (E)$
{-5}	$\overleftrightarrow{EG}\ (G)$
{-6}	$\overleftrightarrow{FC'}\ (F)$
{-7}	$\overleftrightarrow{FC'}\ (C')$
{-8}	$\overleftrightarrow{FG}\ (F)$
{-9}	$\overleftrightarrow{FG}\ (G)$
{-10}	$\overleftrightarrow{A'F}\ (A')$
{-11}	$\overleftrightarrow{A'F}\ (F)$
{-12}	$\overleftrightarrow{A'F}\ (E)$
{-13}	$\overleftrightarrow{FG}\ (C')$
{-14}	$\overleftrightarrow{FC'}\ (E')$
{-15}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating (with copying) the top quantifier in -1 with the terms: F
, E!1
, $\overleftrightarrow{FC'}$
,

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$(F \in \overleftrightarrow{FC'}) \wedge (E' \in \overleftrightarrow{FC'}) \Rightarrow$ $\overleftrightarrow{FC'} = \overleftrightarrow{FE'} \vee F = E'$
{-3}	$(E' \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{E'G} \vee E' = G$
{-4}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-5}	$\overleftrightarrow{EG}\ (E)$
{-6}	$\overleftrightarrow{EG}\ (G)$
{-7}	$\overleftrightarrow{FC'}\ (F)$
{-8}	$\overleftrightarrow{FC'}\ (C')$
{-9}	$\overleftrightarrow{FG}\ (F)$
{-10}	$\overleftrightarrow{FG}\ (G)$
{-11}	$\overleftrightarrow{A'F}\ (A')$
{-12}	$\overleftrightarrow{A'F}\ (F)$
{-13}	$\overleftrightarrow{A'F}\ (E)$
{-14}	$\overleftrightarrow{FG}\ (C')$
{-15}	$\overleftrightarrow{FC'}\ (E')$
{-16}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: F
, C!1
, $\overleftrightarrow{FG}$
,

`SegmentDivision.1.2.2:`

{-1}	$(F \in \overleftrightarrow{FG}) \wedge (C' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{FC'} \vee F = C'$
{-2}	$(F \in \overleftrightarrow{FC'}) \wedge (E' \in \overleftrightarrow{FC'}) \Rightarrow$ $\overleftrightarrow{FC'} = \overleftrightarrow{FE'} \vee F = E'$
{-3}	$(E' \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG}) \Rightarrow$ $\overleftrightarrow{EG} = \overleftrightarrow{E'G} \vee E' = G$
{-4}	$(F \in \overleftrightarrow{A'F}) \wedge (E \in \overleftrightarrow{A'F}) \Rightarrow$ $\overleftrightarrow{A'F} = \overleftrightarrow{FE} \vee F = E$
{-5}	$\overleftrightarrow{EG}\ (E)$
{-6}	$\overleftrightarrow{EG}\ (G)$
{-7}	$\overleftrightarrow{FC'}\ (F)$
{-8}	$\overleftrightarrow{FC'}\ (C')$
{-9}	$\overleftrightarrow{FG}\ (F)$
{-10}	$\overleftrightarrow{FG}\ (G)$
{-11}	$\overleftrightarrow{A'F}\ (A')$
{-12}	$\overleftrightarrow{A'F}\ (F)$
{-13}	$\overleftrightarrow{A'F}\ (E)$
{-14}	$\overleftrightarrow{FG}\ (C')$
{-15}	$\overleftrightarrow{FC'}\ (E')$
{-16}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-2}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-3}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-4}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-5}	$\overleftrightarrow{EG}\ (E)$
{-6}	$\overleftrightarrow{EG}\ (G)$
{-7}	$\overleftrightarrow{FC'}\ (F)$
{-8}	$\overleftrightarrow{FC'}\ (C')$
{-9}	$\overleftrightarrow{FG}\ (F)$
{-10}	$\overleftrightarrow{FG}\ (G)$
{-11}	$\overleftrightarrow{A'F}\ (A')$
{-12}	$\overleftrightarrow{A'F}\ (F)$
{-13}	$\overleftrightarrow{A'F}\ (E)$
{-14}	$\overleftrightarrow{FG}\ (C')$
{-15}	$\overleftrightarrow{FC'}\ (E')$
{-16}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying OnlyLine
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$\overleftrightarrow{FG} = \overleftrightarrow{FC'}$
{-3}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE'}$
{-4}	$\overleftrightarrow{EG} = \overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{A'F} = \overleftrightarrow{FE}$
{-6}	$\overleftrightarrow{EG}\ (E)$
{-7}	$\overleftrightarrow{EG}\ (G)$
{-8}	$\overleftrightarrow{FC'}\ (F)$
{-9}	$\overleftrightarrow{FC'}\ (C')$
{-10}	$\overleftrightarrow{FG}\ (F)$
{-11}	$\overleftrightarrow{FG}\ (G)$
{-12}	$\overleftrightarrow{A'F}\ (A')$
{-13}	$\overleftrightarrow{A'F}\ (F)$
{-14}	$\overleftrightarrow{A'F}\ (E)$
{-15}	$\overleftrightarrow{FG}\ (C')$
{-16}	$\overleftrightarrow{FC'}\ (E')$
{-17}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: G
, E!1
, $\overleftrightarrow{FG}$
,

`SegmentDivision.1.2.2:`

{-1}	$(G \in \overleftrightarrow{FG}) \wedge (E' \in \overleftrightarrow{FG}) \Rightarrow$ $\overleftrightarrow{FG} = \overleftrightarrow{GE'} \vee G = E'$
{-2}	$\overleftrightarrow{FG} = \overleftrightarrow{FC'}$
{-3}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE'}$
{-4}	$\overleftrightarrow{EG} = \overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{A'F} = \overleftrightarrow{FE}$
{-6}	$\overleftrightarrow{EG}\ (E)$
{-7}	$\overleftrightarrow{EG}\ (G)$
{-8}	$\overleftrightarrow{FC'}\ (F)$
{-9}	$\overleftrightarrow{FC'}\ (C')$
{-10}	$\overleftrightarrow{FG}\ (F)$
{-11}	$\overleftrightarrow{FG}\ (G)$
{-12}	$\overleftrightarrow{A'F}\ (A')$
{-13}	$\overleftrightarrow{A'F}\ (F)$
{-14}	$\overleftrightarrow{A'F}\ (E)$
{-15}	$\overleftrightarrow{FG}\ (C')$
{-16}	$\overleftrightarrow{FC'}\ (E')$
{-17}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.1.2.2:`

{-1}	$(\overleftrightarrow{FG}\,(E') \Rightarrow \overleftrightarrow{FG}{=}\overleftrightarrow{GE'})$
{-2}	$\overleftrightarrow{FG}{=}\overleftrightarrow{FC'}$
{-3}	$\overleftrightarrow{FC'}{=}\overleftrightarrow{FE'}$
{-4}	$\overleftrightarrow{EG}{=}\overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{A'F}{=}\overleftrightarrow{FE}$
{-6}	$\overleftrightarrow{EG}\,(E)$
{-7}	$\overleftrightarrow{EG}\,(G)$
{-8}	$\overleftrightarrow{FC'}\,(F)$
{-9}	$\overleftrightarrow{FC'}\,(C')$
{-10}	$\overleftrightarrow{FG}\,(F)$
{-11}	$\overleftrightarrow{FG}\,(G)$
{-12}	$\overleftrightarrow{A'F}\,(A')$
{-13}	$\overleftrightarrow{A'F}\,(F)$
{-14}	$\overleftrightarrow{A'F}\,(E)$
{-15}	$\overleftrightarrow{FG}\,(C')$
{-16}	$\overleftrightarrow{FC'}\,(E')$
{-17}	$\overleftrightarrow{EG}\,(E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\,(E)$
{9}	$A' = C'$

Replacing using formula -2,

`SegmentDivision.1.2.2:`

{-1}	$(\overleftrightarrow{FC'}\ (E') \Rightarrow \overleftrightarrow{FC'}{=}\overleftrightarrow{GE'})$
{-2}	$\overleftrightarrow{FG}{=}\overleftrightarrow{FC'}$
{-3}	$\overleftrightarrow{FC'}{=}\overleftrightarrow{FE'}$
{-4}	$\overleftrightarrow{EG}{=}\overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{A'F}{=}\overleftrightarrow{FE}$
{-6}	$\overleftrightarrow{EG}\ (E)$
{-7}	$\overleftrightarrow{EG}\ (G)$
{-8}	$\overleftrightarrow{FC'}\ (F)$
{-9}	$\overleftrightarrow{FC'}\ (C')$
{-10}	$\overleftrightarrow{FC'}\ (F)$
{-11}	$\overleftrightarrow{FC'}\ (G)$
{-12}	$\overleftrightarrow{A'F}\ (A')$
{-13}	$\overleftrightarrow{A'F}\ (F)$
{-14}	$\overleftrightarrow{A'F}\ (E)$
{-15}	$\overleftrightarrow{FC'}\ (C')$
{-16}	$\overleftrightarrow{FC'}\ (E')$
{-17}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Replacing using formula -16,

`SegmentDivision.1.2.2:`

{-1}	(TRUE $\Rightarrow \overleftrightarrow{FC'}=\overleftrightarrow{GE'}$)
{-2}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-3}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-4}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-6}	$\overleftrightarrow{EG}$ (E)
{-7}	$\overleftrightarrow{EG}$ (G)
{-8}	$\overleftrightarrow{FC'}$ (F)
{-9}	$\overleftrightarrow{FC'}$ (C')
{-10}	$\overleftrightarrow{FC'}$ (F)
{-11}	$\overleftrightarrow{FC'}$ (G)
{-12}	$\overleftrightarrow{A'F}$ (A')
{-13}	$\overleftrightarrow{A'F}$ (F)
{-14}	$\overleftrightarrow{A'F}$ (E)
{-15}	$\overleftrightarrow{FC'}$ (C')
{-16}	$\overleftrightarrow{FC'}$ (E')
{-17}	$\overleftrightarrow{EG}$ (E')
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}$ (E)
{9}	$A' = C'$

Applying propositional simplification and decision procedures,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{FC'}=\overleftrightarrow{GE'}$
{-2}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-3}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-4}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-6}	$\overleftrightarrow{EG}\ (E)$
{-7}	$\overleftrightarrow{EG}\ (G)$
{-8}	$\overleftrightarrow{FC'}\ (F)$
{-9}	$\overleftrightarrow{FC'}\ (C')$
{-10}	$\overleftrightarrow{FC'}\ (F)$
{-11}	$\overleftrightarrow{FC'}\ (G)$
{-12}	$\overleftrightarrow{A'F}\ (A')$
{-13}	$\overleftrightarrow{A'F}\ (F)$
{-14}	$\overleftrightarrow{A'F}\ (E)$
{-15}	$\overleftrightarrow{FC'}\ (C')$
{-16}	$\overleftrightarrow{FC'}\ (E')$
{-17}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying OnlyLine
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$\overleftrightarrow{FC'} = \overleftrightarrow{GE'}$
{-3}	$\overleftrightarrow{FG} = \overleftrightarrow{FC'}$
{-4}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE'}$
{-5}	$\overleftrightarrow{EG} = \overleftrightarrow{E'G}$
{-6}	$\overleftrightarrow{A'F} = \overleftrightarrow{FE}$
{-7}	$\overleftrightarrow{EG}\ (E)$
{-8}	$\overleftrightarrow{EG}\ (G)$
{-9}	$\overleftrightarrow{FC'}\ (F)$
{-10}	$\overleftrightarrow{FC'}\ (C')$
{-11}	$\overleftrightarrow{FC'}\ (F)$
{-12}	$\overleftrightarrow{FC'}\ (G)$
{-13}	$\overleftrightarrow{A'F}\ (A')$
{-14}	$\overleftrightarrow{A'F}\ (F)$
{-15}	$\overleftrightarrow{A'F}\ (E)$
{-16}	$\overleftrightarrow{FC'}\ (C')$
{-17}	$\overleftrightarrow{FC'}\ (E')$
{-18}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: F
, E
, $\overleftrightarrow{FC'}$
,

`SegmentDivision.1.2.2:`

{-1}	$(F \in \overleftrightarrow{FC'}) \wedge (E \in \overleftrightarrow{FC'}) \Rightarrow$ $\overleftrightarrow{FC'} = \overleftrightarrow{FE} \vee F = E$
{-2}	$\overleftrightarrow{FC'} = \overleftrightarrow{GE'}$
{-3}	$\overleftrightarrow{FG} = \overleftrightarrow{FC'}$
{-4}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE'}$
{-5}	$\overleftrightarrow{EG} = \overleftrightarrow{E'G}$
{-6}	$\overleftrightarrow{A'F} = \overleftrightarrow{FE}$
{-7}	$\overleftrightarrow{EG}\ (E)$
{-8}	$\overleftrightarrow{EG}\ (G)$
{-9}	$\overleftrightarrow{FC'}\ (F)$
{-10}	$\overleftrightarrow{FC'}\ (C')$
{-11}	$\overleftrightarrow{FC'}\ (F)$
{-12}	$\overleftrightarrow{FC'}\ (G)$
{-13}	$\overleftrightarrow{A'F}\ (A')$
{-14}	$\overleftrightarrow{A'F}\ (F)$
{-15}	$\overleftrightarrow{A'F}\ (E)$
{-16}	$\overleftrightarrow{FC'}\ (C')$
{-17}	$\overleftrightarrow{FC'}\ (E')$
{-18}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.1.2.2:`

{-1}	$(\overleftrightarrow{FC'}\ (E) \Rightarrow \overleftrightarrow{FC'}=\overleftrightarrow{FE})$
{-2}	$\overleftrightarrow{FC'}=\overleftrightarrow{GE'}$
{-3}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-4}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-5}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-6}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-7}	$\overleftrightarrow{EG}\ (E)$
{-8}	$\overleftrightarrow{EG}\ (G)$
{-9}	$\overleftrightarrow{FC'}\ (F)$
{-10}	$\overleftrightarrow{FC'}\ (C')$
{-11}	$\overleftrightarrow{FC'}\ (F)$
{-12}	$\overleftrightarrow{FC'}\ (G)$
{-13}	$\overleftrightarrow{A'F}\ (A')$
{-14}	$\overleftrightarrow{A'F}\ (F)$
{-15}	$\overleftrightarrow{A'F}\ (E)$
{-16}	$\overleftrightarrow{FC'}\ (C')$
{-17}	$\overleftrightarrow{FC'}\ (E')$
{-18}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying LineSym
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}):\ \overleftrightarrow{AB}=\overleftrightarrow{BA})$
{-2}	$(\overleftrightarrow{FC'}\ (E) \Rightarrow \overleftrightarrow{FC'}=\overleftrightarrow{FE})$
{-3}	$\overleftrightarrow{FC'}=\overleftrightarrow{GE'}$
{-4}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-5}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-6}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-7}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-8}	$\overleftrightarrow{EG}\ (E)$
{-9}	$\overleftrightarrow{EG}\ (G)$
{-10}	$\overleftrightarrow{FC'}\ (F)$
{-11}	$\overleftrightarrow{FC'}\ (C')$
{-12}	$\overleftrightarrow{FC'}\ (F)$
{-13}	$\overleftrightarrow{FC'}\ (G)$
{-14}	$\overleftrightarrow{A'F}\ (A')$
{-15}	$\overleftrightarrow{A'F}\ (F)$
{-16}	$\overleftrightarrow{A'F}\ (E)$
{-17}	$\overleftrightarrow{FC'}\ (C')$
{-18}	$\overleftrightarrow{FC'}\ (E')$
{-19}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: G
, E!1
,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{GE'}=\overleftrightarrow{E'G}$
{-2}	$(\overleftrightarrow{FC'}\ (E) \Rightarrow \overleftrightarrow{FC'}=\overleftrightarrow{FE})$
{-3}	$\overleftrightarrow{FC'}=\overleftrightarrow{GE'}$
{-4}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-5}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-6}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-7}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-8}	$\overleftrightarrow{EG}\ (E)$
{-9}	$\overleftrightarrow{EG}\ (G)$
{-10}	$\overleftrightarrow{FC'}\ (F)$
{-11}	$\overleftrightarrow{FC'}\ (C')$
{-12}	$\overleftrightarrow{FC'}\ (F)$
{-13}	$\overleftrightarrow{FC'}\ (G)$
{-14}	$\overleftrightarrow{A'F}\ (A')$
{-15}	$\overleftrightarrow{A'F}\ (F)$
{-16}	$\overleftrightarrow{A'F}\ (E)$
{-17}	$\overleftrightarrow{FC'}\ (C')$
{-18}	$\overleftrightarrow{FC'}\ (E')$
{-19}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Replacing using formula -1,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{GE'}=\overleftrightarrow{E'G}$
{-2}	$(\overleftrightarrow{FC'}\ (E) \Rightarrow \overleftrightarrow{FC'}=\overleftrightarrow{FE})$
{-3}	$\overleftrightarrow{FC'}=\overleftrightarrow{E'G}$
{-4}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-5}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-6}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-7}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-8}	$\overleftrightarrow{EG}\ (E)$
{-9}	$\overleftrightarrow{EG}\ (G)$
{-10}	$\overleftrightarrow{FC'}\ (F)$
{-11}	$\overleftrightarrow{FC'}\ (C')$
{-12}	$\overleftrightarrow{FC'}\ (F)$
{-13}	$\overleftrightarrow{FC'}\ (G)$
{-14}	$\overleftrightarrow{A'F}\ (A')$
{-15}	$\overleftrightarrow{A'F}\ (F)$
{-16}	$\overleftrightarrow{A'F}\ (E)$
{-17}	$\overleftrightarrow{FC'}\ (C')$
{-18}	$\overleftrightarrow{FC'}\ (E')$
{-19}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying propositional simplification and decision procedures,

`SegmentDivision.1.2.2:`

{-1}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE}$
{-2}	$\overleftrightarrow{GE'}=\overleftrightarrow{E'G}$
{-3}	$\overleftrightarrow{FC'}=\overleftrightarrow{E'G}$
{-4}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-5}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-6}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-7}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-8}	$\overleftrightarrow{EG}\ (E)$
{-9}	$\overleftrightarrow{EG}\ (G)$
{-10}	$\overleftrightarrow{FC'}\ (F)$
{-11}	$\overleftrightarrow{FC'}\ (C')$
{-12}	$\overleftrightarrow{FC'}\ (F)$
{-13}	$\overleftrightarrow{FC'}\ (G)$
{-14}	$\overleftrightarrow{A'F}\ (A')$
{-15}	$\overleftrightarrow{A'F}\ (F)$
{-16}	$\overleftrightarrow{A'F}\ (E)$
{-17}	$\overleftrightarrow{FC'}\ (C')$
{-18}	$\overleftrightarrow{FC'}\ (E')$
{-19}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Applying OnlyLine
where

`SegmentDivision.1.2.2:`

{-1}	$(\forall\,(A, B:\ \text{Point}, l:\ \text{Line}):$ $(A \in l) \wedge (B \in l) \Rightarrow$ $l = \overleftrightarrow{AB} \vee A = B)$
{-2}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE}$
{-3}	$\overleftrightarrow{GE'} = \overleftrightarrow{E'G}$
{-4}	$\overleftrightarrow{FC'} = \overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{FG} = \overleftrightarrow{FC'}$
{-6}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE'}$
{-7}	$\overleftrightarrow{EG} = \overleftrightarrow{E'G}$
{-8}	$\overleftrightarrow{A'F} = \overleftrightarrow{FE}$
{-9}	$\overleftrightarrow{EG}\ (E)$
{-10}	$\overleftrightarrow{EG}\ (G)$
{-11}	$\overleftrightarrow{FC'}\ (F)$
{-12}	$\overleftrightarrow{FC'}\ (C')$
{-13}	$\overleftrightarrow{FC'}\ (F)$
{-14}	$\overleftrightarrow{FC'}\ (G)$
{-15}	$\overleftrightarrow{A'F}\ (A')$
{-16}	$\overleftrightarrow{A'F}\ (F)$
{-17}	$\overleftrightarrow{A'F}\ (E)$
{-18}	$\overleftrightarrow{FC'}\ (C')$
{-19}	$\overleftrightarrow{FC'}\ (E')$
{-20}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Instantiating the top quantifier in -1 with the terms: A!1
, C!1
, $\overleftrightarrow{FC'}$
,

`SegmentDivision.1.2.2:`

{-1}	$(A' \in \overleftrightarrow{FC'}) \wedge (C' \in \overleftrightarrow{FC'}) \Rightarrow$ $\overleftrightarrow{FC'} = \overleftrightarrow{A'C'} \vee A' = C'$
{-2}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE}$
{-3}	$\overleftrightarrow{GE'} = \overleftrightarrow{E'G}$
{-4}	$\overleftrightarrow{FC'} = \overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{FG} = \overleftrightarrow{FC'}$
{-6}	$\overleftrightarrow{FC'} = \overleftrightarrow{FE'}$
{-7}	$\overleftrightarrow{EG} = \overleftrightarrow{E'G}$
{-8}	$\overleftrightarrow{A'F} = \overleftrightarrow{FE}$
{-9}	$\overleftrightarrow{EG}\ (E)$
{-10}	$\overleftrightarrow{EG}\ (G)$
{-11}	$\overleftrightarrow{FC'}\ (F)$
{-12}	$\overleftrightarrow{FC'}\ (C')$
{-13}	$\overleftrightarrow{FC'}\ (F)$
{-14}	$\overleftrightarrow{FC'}\ (G)$
{-15}	$\overleftrightarrow{A'F}\ (A')$
{-16}	$\overleftrightarrow{A'F}\ (F)$
{-17}	$\overleftrightarrow{A'F}\ (E)$
{-18}	$\overleftrightarrow{FC'}\ (C')$
{-19}	$\overleftrightarrow{FC'}\ (E')$
{-20}	$\overleftrightarrow{EG}\ (E')$
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}\ (E)$
{9}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.1.2.2:`

{-1}	FALSE
{-2}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE}$
{-3}	$\overleftrightarrow{GE'}=\overleftrightarrow{E'G}$
{-4}	$\overleftrightarrow{FC'}=\overleftrightarrow{E'G}$
{-5}	$\overleftrightarrow{FG}=\overleftrightarrow{FC'}$
{-6}	$\overleftrightarrow{FC'}=\overleftrightarrow{FE'}$
{-7}	$\overleftrightarrow{EG}=\overleftrightarrow{E'G}$
{-8}	$\overleftrightarrow{A'F}=\overleftrightarrow{FE}$
{-9}	$\overleftrightarrow{EG}$ (E)
{-10}	$\overleftrightarrow{EG}$ (G)
{-11}	$\overleftrightarrow{FC'}$ (F)
{-12}	$\overleftrightarrow{FC'}$ (C')
{-13}	$\overleftrightarrow{FC'}$ (F)
{-14}	$\overleftrightarrow{FC'}$ (G)
{-15}	$\overleftrightarrow{A'F}$ (A')
{-16}	$\overleftrightarrow{A'F}$ (F)
{-17}	$\overleftrightarrow{A'F}$ (E)
{-18}	$\overleftrightarrow{FC'}$ (C')
{-19}	$\overleftrightarrow{FC'}$ (E')
{-20}	$\overleftrightarrow{EG}$ (E')
{1}	$E' = G$
{2}	$E' = A'$
{3}	$E' = F$
{4}	$G = C'$
{5}	$G = F$
{6}	$F = E$
{7}	$F = A'$
{8}	$\overleftrightarrow{A'C'}$ (E)
{9}	$A' = C'$

which is trivially true.

This completes the proof of `SegmentDivision.1.2.2`.

`SegmentDivision.2:`

{-1}	$(C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F$
{-2}	$(E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'$
{1}	$(\exists\, (D:\ \text{Point}):$ $(D \in (\overline{A'F} \cap \overleftrightarrow{EG})) \wedge$ $D \neq A' \wedge D \neq F)$
{2}	$(E \in \overleftrightarrow{A'C'})$
{3}	$A' \neq C' \Rightarrow$ $\exists (D:\ \text{Point}):$ $D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})$

Instantiating the top quantifier in 1 with the terms: E

,

`SegmentDivision.2:`

{-1}	$(C' \in \overline{FG}) \wedge G \neq C' \wedge G \neq F$
{-2}	$(E \in \overline{A'F}) \wedge F \neq E \wedge F \neq A'$
{1}	$(E \in (\overline{A'F} \cap \overleftrightarrow{EG})) \wedge$ $E \neq A' \wedge E \neq F$
{2}	$(E \in \overleftrightarrow{A'C'})$
{3}	$A' \neq C' \Rightarrow$ $\exists (D:\ \text{Point}):$ $D \neq A' \wedge D \neq C' \wedge (D \in \overline{A'C'})$

Trying repeated skolemization, instantiation, and if-lifting,

we get 3 subgoals:

`SegmentDivision.2.1:`

{-1}	$E = A'$
{-2}	$\overline{FG}(C')$
{-3}	$\overline{A'F}(A')$
{1}	$G = C'$
{2}	$G = F$
{3}	$F = A'$
{4}	$\overleftrightarrow{A'C'}\ (A')$
{5}	$A' = C'$

Applying LineExists
where A
gets A!1
, B
gets C!1
,

`SegmentDivision.2.1:`

{-1}	$(A' \in \overleftrightarrow{A'C'}) \wedge (C' \in \overleftrightarrow{A'C'})$
{-2}	$E = A'$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(A')$
{1}	$G = C'$
{2}	$G = F$
{3}	$F = A'$
{4}	$\overleftrightarrow{A'C'}\ (A')$
{5}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.2.1:`

{-1}	FALSE
{-2}	$E = A'$
{-3}	$\overline{FG}(C')$
{-4}	$\overline{A'F}(A')$
{1}	$G = C'$
{2}	$G = F$
{3}	$F = A'$
{4}	$\overleftrightarrow{A'C'}\ (A')$
{5}	$A' = C'$

which is trivially true.

This completes the proof of `SegmentDivision.2.1`.

`SegmentDivision.2.2:`

{-1}	$\overline{C'G}(C')$
{-2}	$\overline{A'C'}(E)$
{-3}	$(F = C')$
{1}	$\overleftrightarrow{EG}\ (E)$
{2}	$G = C'$
{3}	$C' = E$
{4}	$C' = A'$
{5}	$\overleftrightarrow{A'C'}\ (E)$
{6}	$A' = C'$

Applying LineExists
where A
gets E
, B
gets G
,

`SegmentDivision.2.2:`

{-1}	$(E \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG})$
{-2}	$\overline{C'G}(C')$
{-3}	$\overline{A'C'}(E)$
{-4}	$(F = C')$
{1}	$\overleftrightarrow{EG}\ (E)$
{2}	$G = C'$
{3}	$C' = E$
{4}	$C' = A'$
{5}	$\overleftrightarrow{A'C'}\ (E)$
{6}	$A' = C'$

Expanding the definition of member

,

`SegmentDivision.2.2:`

{-1}	FALSE
{-2}	$\overline{C'G}(C')$
{-3}	$\overline{A'C'}(E)$
{-4}	$(F = C')$
{1}	$\overleftrightarrow{EG}\ (E)$
{2}	$G = C'$
{3}	$C' = E$
{4}	$C' = A'$
{5}	$\overleftrightarrow{A'C'}\ (E)$
{6}	$A' = C'$

which is trivially true.

This completes the proof of `SegmentDivision.2.2`.

`SegmentDivision.2.3:`

{-1}	$\overline{FG}(C')$
{-2}	$\overline{A'F}(E)$
{1}	$\overleftrightarrow{EG}\ (E)$
{2}	$G = C'$
{3}	$G = F$
{4}	$F = E$
{5}	$F = A'$
{6}	$\overleftrightarrow{A'C'}\ (E)$
{7}	$A' = C'$
{8}	$\overline{A'C'}(F)$

Applying LineExists
where A
gets E

, B
gets G
,

`SegmentDivision.2.3:`

{-1}	$(E \in \overleftrightarrow{EG}) \wedge (G \in \overleftrightarrow{EG})$
{-2}	$\overline{FG}(C')$
{-3}	$\overline{A'F}(E)$
{1}	$\overleftrightarrow{EG}\ (E)$
{2}	$G = C'$
{3}	$G = F$
{4}	$F = E$
{5}	$F = A'$
{6}	$\overleftrightarrow{A'C'}\ (E)$
{7}	$A' = C'$
{8}	$\overline{A'C'}(F)$

Expanding the definition of member
,

`SegmentDivision.2.3:`

{-1}	FALSE
{-2}	$\overline{FG}(C')$
{-3}	$\overline{A'F}(E)$
{1}	$\overleftrightarrow{EG}\ (E)$
{2}	$G = C'$
{3}	$G = F$
{4}	$F = E$
{5}	$F = A'$
{6}	$\overleftrightarrow{A'C'}\ (E)$
{7}	$A' = C'$
{8}	$\overline{A'C'}(F)$

which is trivially true.

This completes the proof of `SegmentDivision.2.3`.

Q.E.D.