Representing Portfolios

On the representation of trading strategies and financial portfolios

Stefan Dirnstorfer

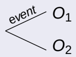
Technical University Munich

2004-Dec-13

Three Operators To Rule The Portfolio

All trading strategies and financial products can be described in terms of three operators.

List of operators

	Initial values
$\ominus$	An inactive time step
$T_c^{\mathbf{n}}$	Transaction of **n units** onto account *c*
	Decision between option O_1 and O_2
	Final valuation

Portfolio Example: A Bond Deal

A multiperiod strategy can be written by a chronologically ordered operator sequence.

$$T_c^{-90} \ominus T_c^{100}$$

Read:

1. Withdraw 90 from account *c*
2. Wait one period
3. receive 100

Mathematica Code:

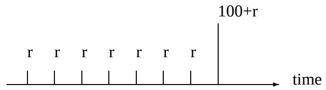
```
Function[V_, T[c, -90, Theta[ T[c, 100, V ]]]]
```

Portfolio Example: A Coupon Bond

Repeated actions can be denoted by the operator power.

$$\left(\ominus T_c^r\right)^M T_c^{100}$$

The operator term reads chronologically from left to right. The term in parenthesis is repeated *M* times.



Mathematica code

```
Function[V_,
Nest[ Function[f_, Theta[T[c, r, f] ]],  T[c, 100, V], M]]
```

Portfolio Example: European Option

The European option offers the the choice between product A and B, after time to maturity M.

$$\Theta^M \left\langle \begin{array}{l} \overset{\max}{} A \\ B \end{array} \right. := \Theta^M \max(A, B)$$

Mathematica code

```
Function[V_,
Nest[Theta,
  IfThen[ A[V]>B[V], A[V], B[V]],
M]]
```

Portfolio Example: American Option

With American option there is a perpetual right to choose X.

$$\lim_{\Delta t \to 0} \left(\Theta^{\Delta t} \left\langle \begin{array}{l} \overset{\max}{} X \\ Id \end{array} \right. \right)^{\frac{M}{\Delta t}}$$

Mathematica code for $\Delta t = 1$

```
Function[V_,
Nest[
Function[f_,
  Theta[ IfThen[X[V]>f, X[V], f]]],
V,M]]
```

Transfer operator

The transfer or shift operator T transfers a deterministic amount onto variable x.

$$T_x^n f(x) := f(x+n)$$

This operation replaces every instance of x with $x + n$.

Mathematica code

```
T[index_, power_, V_] := V /. index-> index+power
```

Process Operator

With Θ we can look one step into the future and evaluate the expectation of a function f under the future process state.

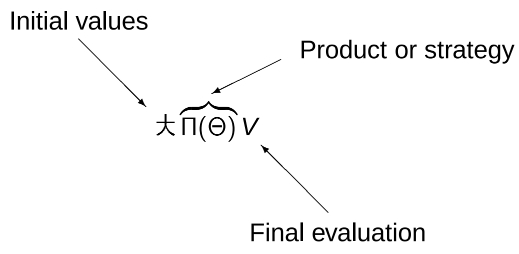
$$\Theta f(x) := \mathbb{E}\big(f(X_{t+1}) \big| X_t = x\big)$$

Example:
Different process scenarios can be combined linearly.

$$\Theta = \underbrace{T_c^{rc}}_{\text{pay interest}} \underbrace{\left\langle \begin{array}{l} \overset{p}{} \; T_S^{+S} \\ \underset{1-p}{} \; T_S^{-\frac{1}{2}S} \end{array} \right.}_{\text{Vary } S}$$

Evaluation Sequence

We need to define initial process variables and determine the property of interest.

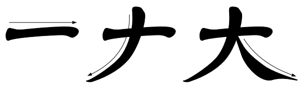


Evaluation Operator

The chronological operator order is maintained by a new operator 大 that applies initial values from the left hand side.

$$大 V \;=\; \underset{X=X_0}{大}\, V \;:=\; V|_{X=X_0}$$

How to write:



Evaluation Of S

Starting with $S = 100$, what is S's expected value after one period?

$$\underset{S=100}{大}\, \Theta\, S \;=\; 200p + 50(1-p)$$

There exists a pseudo probability p, such that discounted S is a martingale.

$$\exists p: \quad \underset{r=1/9}{大}\, \Theta\, S \;=\; S(1+r)$$

The result is found easily:

$$p = \frac{11}{27}$$

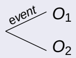
Pricing A Product

$$\begin{aligned}
\underset{\substack{c=0\\S=100}}{大}\, \underbrace{T_c^{-S}}_{\text{buy stock}}\, \underbrace{\Theta}_{\text{wait}}\, \underbrace{T_c^{+S}}_{\text{sell stock}}\, \underbrace{c}_{\text{profit?}} &= 大 T_c^{-S} \Theta\, c + S \\
&= 大 T_c^{-S} \frac{10(c+S)}{9} \\
&= 大 \frac{10c}{9} \quad = \quad 0 \\
\underset{\substack{c=0\\S=100}}{大}\, \underbrace{T_c^{-S}\, \Theta\, T_c^{+S}}_{\Pi(\Theta)}\, \underbrace{c^2}_{\text{risk?}} &= 5432 \quad = \quad (73.7)^2
\end{aligned}$$

Hedgeable price:	0€
Standard deviation:	73.7€

Five Operators To Rule Them All

List of operators

大	Initial values
⊖	An inactive time step
T_c	Transaction onto account c
	Decision between option O_1 and O_2
V	Final valuation