1 Dadim implementation

1.1 Multiscale operators

```
> # Licensed under the Gnu Public License
> da:=(dadim)-> dadim[1];
> trans:=(dir, len, dadim)-> dadim[2](dir, len);
> zoom:=(dir, dadim)-> dadim[3](dir);
```

$$da := dadim \to dadim_1$$

$$trans := (dir,\, len,\, dadim) \to dadim_2(dir,\, len)$$

$$zoom := (dir,\, dadim) \to dadim_3(dir)$$

1.2 Constant

```
> cons:=c->[c, (dir, len) -> cons(c), (dir) -> cons(c)];
```

$$cons := c \to [c,\, (dir,\, len) \to \mathrm{cons}(c),\, dir \to \mathrm{cons}(c)]$$

```
> da(cons(5));
```

$$5$$

```
> da(cons("Hallo World"));
```

$$\text{“Hallo World”}$$

1.3 Variable

1.3.1 Definition

```
> x:=(i)->[
> 0,
> (dir,len)->
> piecewise(dir=i,
> x(i) + cons(len),
> x(i)),
> (dir)->
> piecewise(dir=i,
> x(i) / 2,
> x(i))
> ];
```

$$x := i \to [0,\, (dir,\, len) \to \mathrm{piecewise}(dir = i,\, \mathrm{x}(i) + \mathrm{cons}(len),\, \mathrm{x}(i)),$$
$$dir \to \mathrm{piecewise}(dir = i,\, \frac{1}{2}\,\mathrm{x}(i),\, \mathrm{x}(i))]$$

1.3.2 Examples

```
> da(trans(0,1,x(0)));
```

$$1$$

```
> da(trans(1,1,x(0)));
```

$$0$$

```
> da(trans(0,3,zoom(0,x(0))));
```

$$\frac{3}{2}$$

```
> seq(da(trans(0,i,zoom(0,x(0)))), i=-3..3);
```

$$\frac{-3}{2},\ -1,\ \frac{-1}{2},\ 0,\ \frac{1}{2},\ 1,\ \frac{3}{2}$$

1.4 Nested Functions

```
> multop:=(arg,F)->[
> F(map(da, arg)),
> (dir,len)->multop(map(a->trans(dir, len,a), arg),F),
> (dir)->multop(map(a->zoom(dir,a), arg), F)];
```

$$\begin{aligned}&\mathit{multop} := (\mathit{arg},\, F) \to [F(\mathrm{map}(\mathit{da},\, \mathit{arg})),\\&(\mathit{dir},\, \mathit{len}) \to \mathrm{multop}(\mathrm{map}(a \to \mathrm{trans}(\mathit{dir},\, \mathit{len},\, a),\, \mathit{arg}),\, F),\\&\mathit{dir} \to \mathrm{multop}(\mathrm{map}(a \to \mathrm{zoom}(\mathit{dir},\, a),\, \mathit{arg}),\, F)]\end{aligned}$$

```
> mult:=(dadim1,dadim2)->multop([dadim1, dadim2], x->x[1]*x[2]);
```

$$\mathit{mult} := (\mathit{dadim1},\, \mathit{dadim2}) \to \mathrm{multop}([\mathit{dadim1},\, \mathit{dadim2}],\, x \to x_1\, x_2)$$

```
> da(mult(cons(3),cons(4)));
```

$$12$$

```
> div:=(dadim1,dadim2)->multop([dadim1, dadim2], x->x[1]/x[2]);
```

$$\mathit{div} := (\mathit{dadim1},\, \mathit{dadim2}) \to \mathrm{multop}([\mathit{dadim1},\, \mathit{dadim2}],\, x \to \frac{x_1}{x_2})$$

```
> da(div(cons(3),cons(4)));
```

$$\frac{3}{4}$$

```
> lg:=dadim->multop([dadim],x->ln(x[1]));
```

$$\mathit{lg} := \mathit{dadim} \to \mathrm{multop}([\mathit{dadim}],\, x \to \ln(x_1))$$

```
> da(lg(cons(2)));
```

$$\ln(2)$$

2 Basic calculus

2.1 Derivative

2.1.1 Definition

```
> dif:=(i,dadim) -> [
> da(trans(i,1,dadim)) - da(dadim),
> (dir, len) ->
> dif(i,trans(dir,len,dadim)),
> (dir) ->
> dif(i,zoom(dir,dadim))
> ];
```

$$\begin{aligned}&\mathit{dif} := (i,\, \mathit{dadim}) \to [\mathrm{da}(\mathrm{trans}(i,\, 1,\, \mathit{dadim})) - \mathrm{da}(\mathit{dadim}),\\&(\mathit{dir},\, \mathit{len}) \to \mathrm{dif}(i,\, \mathrm{trans}(\mathit{dir},\, \mathit{len},\, \mathit{dadim})),\ \mathit{dir} \to \mathrm{dif}(i,\, \mathrm{zoom}(\mathit{dir},\, \mathit{dadim}))]\end{aligned}$$

2.1.2 Examples

```
> seq(da(trans(0,i,dif(0,mult(x(0),x(0))))),i=-3..3);
```

$$-5,\ -3,\ -1,\ 1,\ 3,\ 5,\ 7$$

```
> heaviside:=i->multop([x(i)], x-> piecewise(x[1]>0,1,0));
```

$$\mathit{heaviside} := i \to \mathrm{multop}([\mathrm{x}(i)],\, x \to \mathrm{piecewise}(0 < x_1,\, 1,\, 0))$$

```
> seq(da(trans(0,i,heaviside(0))),i=-3..3);
```

$$0,\, 0,\, 0,\, 0,\, 1,\, 1,\, 1$$

```
> delta:=i->div(dif(i,heaviside(i)),dif(i,x(i)));
```

$$\delta := i \to \mathrm{div}(\mathrm{dif}(i,\, \mathrm{heaviside}(i)),\, \mathrm{dif}(i,\, \mathrm{x}(i)))$$

```
> seq(da(trans(0,i,delta(0))),i=-3..3);
```

$$0,\, 0,\, 0,\, 1,\, 0,\, 0,\, 0$$

```
> seq(da(trans(0,i,zoom(0,delta(0)))),i=-3..3);
```

$$0,\, 0,\, 0,\, 2,\, 0,\, 0,\, 0$$

2.1.3 Delta function

```
> ifgz:= (test, dadim1, dadim2) ->
> multop([test,dadim1,dadim2],
> x -> if x[1]>0 then x[2] else x[3] end if):
> delta:= i-> div(dif(i,ifgz(x(i),cons(1),cons(0))),dif(i,x(i)));
```

$$\delta := i \to \mathrm{div}(\mathrm{dif}(i,\, \mathrm{ifgz}(\mathrm{x}(i),\, \mathrm{cons}(1),\, \mathrm{cons}(0))),\, \mathrm{dif}(i,\, \mathrm{x}(i)))$$

```
> seq(da(trans(0,i, zoom(0,zoom(0, delta(0))))), i=-3..3);
```

$$0,\, 0,\, 0,\, 4,\, 0,\, 0,\, 0$$

```
> da(delta(0));
```

$$1$$

```
> da(zoom(1,delta(1)));
```

$$2$$

2.1.4 Complete differential (failed)

```
> dif2:=proc(dadim, n)
> local S,i;
> S:=dif(0,dadim);
> for i from 1 to n do
> S:=S+dif(i,dadim)
> end do; S; end proc:
> diftest:=div(dif2(x(0)+mult(x(1),x(1)),1), dif(0,x(0))):
> for j from 0 to 5 do
> diftest=zoom(0,diftest);
> diftest=zoom(1,diftest);
> end do:
> da(diftest);
```

$$2$$

2.2 Haar Wavelet

```
>  haar:=(i,dadim)->[
>  da(dadim),
>  (dir, len) ->
>  haar(i,trans(dir,len,dadim)),
>  (dir) ->
>  piecewise(dir=i,
>  haar(i,zoom(dir,dadim) + trans(i, 1, zoom(dir, dadim,dir))),
>  haar(i,zoom(dir,dadim)))
>  ];
```

$haar := (i,\ dadim) \rightarrow [\mathrm{da}(dadim),\ (dir,\ len) \rightarrow \mathrm{haar}(i,\ \mathrm{trans}(dir,\ len,\ dadim)),\ dir \rightarrow$ $\mathrm{piecewise}(dir = i,\ \mathrm{haar}(i,\ \mathrm{zoom}(dir,\ dadim) + \mathrm{trans}(i,\ 1,\ \mathrm{zoom}(dir,\ dadim,\ dir))),$ $\mathrm{haar}(i,\ \mathrm{zoom}(dir,\ dadim)))]$

```
>  B:=zoom(0,zoom(0,haar(0,mult(
>  haar(0,mult(delta(0),dif(0,x(0)))),dif(0,x(0)))))):
>  seq(da(trans(0,i,B)),i=-8..2);
```

$$0,\ 0,\ \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1,\ \frac{3}{4},\ \frac{1}{2},\ \frac{1}{4},\ 0,\ 0$$

2.3 Integral

```
>  integ:=(i,dadim)->[
>  0,
>  (dir, len)->
>  piecewise(dir=i,
>  piecewise(
>  len>0, trans(dir,len-1,
>  integ(i,dadim) + haar(i,dadim)),
>  len<0, trans(dir, len+1,
>  integ(i,dadim) -
>  trans(dir,-1,haar(i,dadim))),
>  len=0, integ(i,dadim)),
>  integ(i,trans(dir,len,dadim))),
>  (dir) ->
>  integ(i,zoom(dir,dadim))
>  ]:
>  F:=zoom(0,integ(0,dif(0,x(0)))):
>  seq(da(trans(0,i,F)),i=-3..10);
```

$$\frac{-3}{2},\ -1,\ \frac{-1}{2},\ 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2,\ \frac{5}{2},\ 3,\ \frac{7}{2},\ 4,\ \frac{9}{2},\ 5$$

2.4 Solver

```
>  solver:= (i, dadim) -> [
>  -da(dadim)/(da(trans(i,1,dadim))-da(dadim)),
>  (dir,len) ->
>  solver(i, trans(dir,len,dadim)),
>  (dir) ->
>  piecewise(dir=i,
>  ifgz(solver(i,trans(i,1,zoom(i,dadim))),
>  cons(1/2) + 1/2 * solver(i,trans(i,1,zoom(i,dadim))),
>  1/2 * solver(i,zoom(i,dadim))),
>  solver(i,zoom(dir,dadim)))
>  ]:
```

```
>  a:=1/2:
>  sqrt2:=solver(0,mult(x(0),x(0))-cons(a)):
>  for i from 0 to 5 do
>  print ([da(sqrt2),evalf(da(sqrt2)-sqrt(a))]);
>  sqrt2:=zoom(0,sqrt2);
>  end do:
```

$$[\frac{1}{2}, -0.2071067810]$$

$$[\frac{2}{3}, -0.0404401143]$$

$$[\frac{7}{10}, -0.0071067810]$$

$$[\frac{31}{44}, -0.0025613265]$$

$$[\frac{65}{92}, -0.0005850419]$$

$$[\frac{509}{720}, -0.0001623366]$$

2.5 Convolution

2.5.1 Shift operator

```
>  shift:=(i,field,dadim) -> [
>  (da(field)-floor(da(field)))*
>  da(trans(i,ceil(da(field)),dadim))+
>  (1-da(field)+floor(da(field)))*
>  da(trans(i,floor(da(field)),dadim)),
>  (dir,len)->
>  shift(i,trans(dir,len,field),trans(dir,len,dadim)),
>  (dir)->
>  piecewise(dir=i,
>  shift(i,2*zoom(dir,field),zoom(dir,dadim)),
>  shift(i,zoom(dir,field),zoom(dir,dadim)))
>  ]:
```

2.5.2 Example

```
> da(trans(0,3,zoom(0,shift(0,cons(0),x(0)))));
```

$$\frac{3}{2}$$

2.5.3 Convolution

```
> Phi:=lg(cons(1)-x(1)):
> F:=delta(0):
> F:=mult(haar(1,shift(0,Phi,F)),dif(1,x(1))):

> F:=zoom(1,F):
> F:=zoom(0,F):


> seq(da(trans(0,j,F)),j=-2..10);
```

$0,\ 0,\ 1,\ 4\ln(\frac{3}{4})+2,\ 2-4\ln(\frac{3}{4})-4\ln(2),\ -2+4\ln(2),\ 0,\ -4\ln(4)+6,\ -5+4\ln(4),\ 0,\ 0,\ 0,$
0

```
> evalf(%);
```

0., 0., 1., 0.849271710, 0.378139568, 0.772588722, 0., 0.454822556, 0.545177444, 0., 0., 0., 0.