Referee's Report on "Pattern in Pi" by Stefan Dirnstorfer

November 19, 2004

This referee continues to have reservations about the submitted manuscript. Generally speaking, it is difficult to submit any particular chapter of a thesis as a self–contained paper, although it is possible. More usually, an advisor recommends that a Ph D student carefully rewrite the work as an independent article for publication. It seems to this referee that Dirnstorfer could benefit from this advice. What follows is more detail about the criticisms at this stage of the author's developmemt.

- There is a serious overstatement on the role of the classical 1973 formula of Black and Scholes appearing on page 5.

  > "The great achievement of Black and Scholes was that options have to be priced with their model ..."

  This is innaccurate. The Black/Scholes "miracle" is that when the process is governed by geometric Brownian motion, evaluating a call option depends only on the spot rate and the variance under the risk neutral measure where now the drift is irrelevant. As pointed out in Carr & Wu, [1], this by now classical Brownian motion benchmark is not enough because there are at least 3 systematic and persistent departures for both the statistical and risk neutral processes. Carr & Wu employ time–changed Lévy processes as a way to simultaneously and parsimoniously capture all three departures. So, clearly, *options <u>do</u> <u>not</u> have to be priced with their model.*

- Lines 5–6 from the top of page 10 illustrates a type of incompleteness of the manuscript, where it is stated that a key (to this referee) numerical scheme "is discussed in a separate paper". Perhaps the numerical scheme appears in another chapter of the author's thesis.

- It is well known that geometric Brownian motion can be approximately arbitrarily closely by a fine binomial tree. It is easy to obtain the Martingale probabilities for each stage of such trees, for example, by linear programming and the associated optimal dual variables. In the simplest of all cases, namely just one step it is easiest to find the unique, arbitrage–free primal–dual solution. Consider the simple example of Ross [2], where the strike price is 150.

Example in the earlier Ross edition to [2]

Figure 5.1 p62 gives the tree for a simple call option, leading to the following LP problem.

$$\begin{array}{llcll} v_P = & min\ 100x_O & + & x_D & \\ dual\ vars & & & & \\ \frac{1+2r}{3(1+r)} & 200x_O & + & (1+r)x_D & \geq max\,\{200-150,0\} \\ \frac{2-2r}{3(1+r)} & 50x_O & + & (1+r)x_D & \geq max\,\{50-150,0\} \\ solution & x_O^* = \frac{1}{3} & & \frac{-50x_O^*}{1+r} & v_P = \frac{50+100r}{3(1+r)} \end{array} \tag{1}$$

We simply verify nonarbitrage when,

$$p_b = 50 < (1+r)100 < p_g = 200 \tag{2}$$

Using LP we compute the value of the call option to be $v_P = \left(100 - \frac{50}{1+r}\right)\frac{1}{3}$.

This is very specific, exact, and arbitrage–free. Any operator formalism must be able to accurately solve the most elementary of the laws of motion, namely a one–step binomial tree.

Contrast this to the author's **5 Example**, page 9. There we read

> "Our trader has an obligation to her customers (correcting the misspelling) in the form of a call option with strike 10. Her mission is to meet her obligations in either in cash or stocks with a minimum squared distance."

This is simply wrong because the Martingales are the key as illustrated in the Ross example. Otherwise, there are opportunities for arbitrage. It is not clear, as stated in the first report that arbitrage can be avoided.

Maybe when Dirnstorfer's formalism's are applied to the Ross example, he will be led to the unique optimal solution, which a priori has nothing to do with any kind of distance measure. This referee has not been able to do this with Dirnstorfer's formalisms, but this could be due to the omission of details on the numerical scheme.

It is premature to accept this paper for publication or presentation.

References

[1] P. Carr and L. Wu. Time-changed Lévy processes and option pricing. *J. Financialo Economics*, 71:113–141, 2004.

[2] S. M. Ross. *An Elementary Introduction to Mathematical Finance Options and Other Topics*. Cambridge University Press, 40 West 20^{th} Street, New York, NY 1001-4211, 2003. Second Edition.